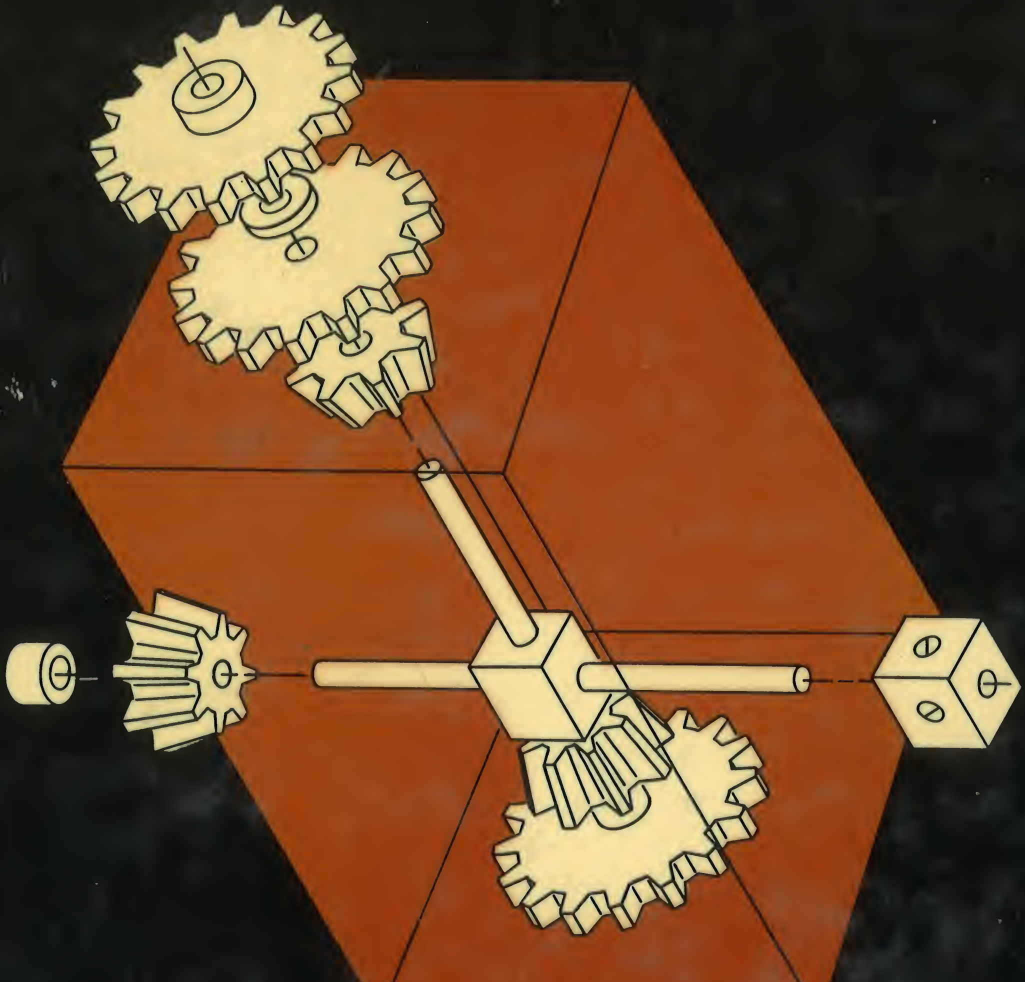
MECHANISMS

MACHINES



Electromechanical Technology Series

TERC EMT STAFF

DELMAR PUBLISHERS, MOUNTAINVIEW AVENUE, ALBANY, NEW YORK 12205

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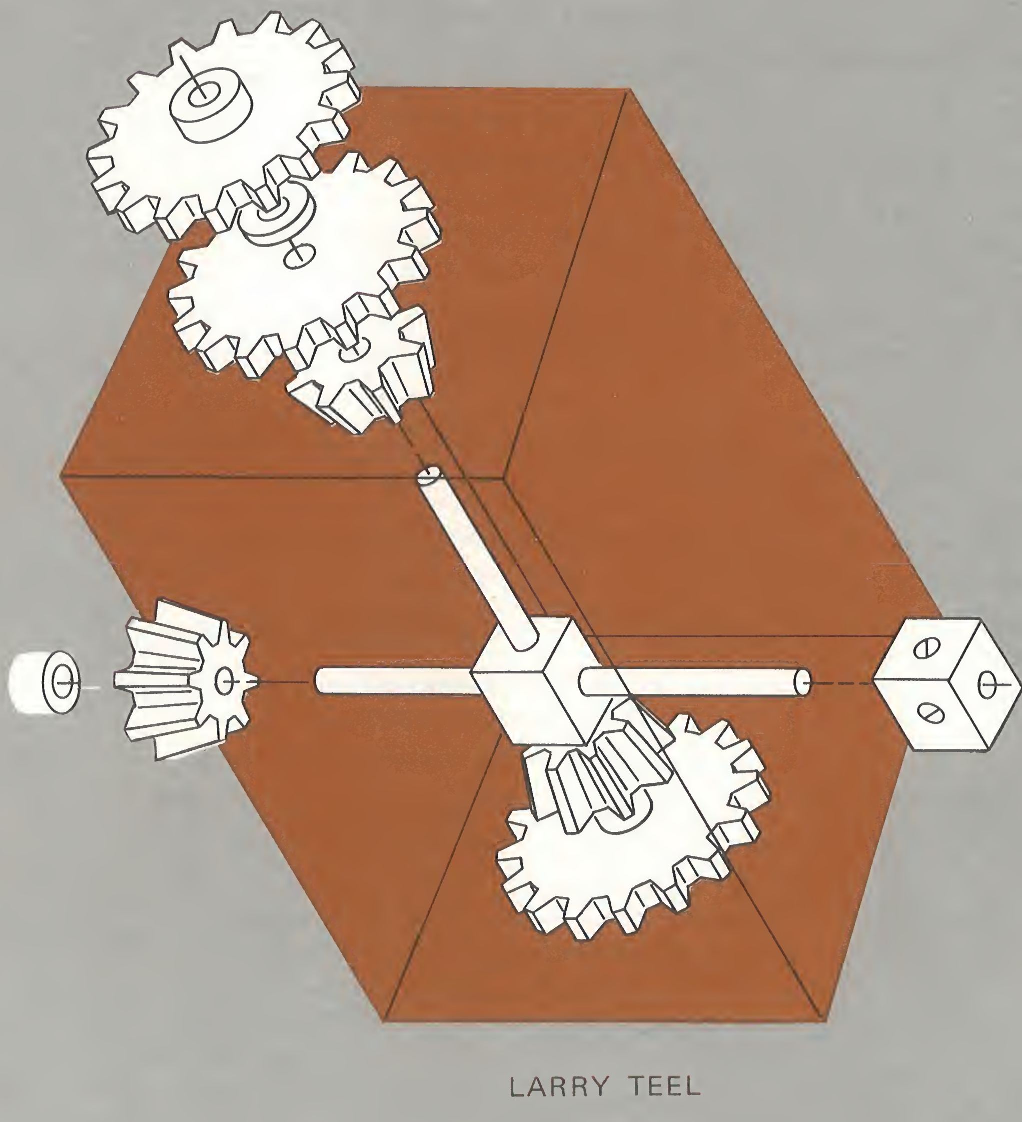

LARRY TEEL

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Foreword

The marriage of electronics and technology is creating new demands for technical personnel in today's industries. New occupations have emerged with combination skill requirements well beyond the capability of many technical specialists. Increasingly, technicians who work with systems and devices of many kinds — mechanical, hydraulic, pneumatic, thermal, and optical — must be competent also in electronics. This need for combination skills is especially significant for the youngster who is preparing for a career in industrial technology.

This manual is one of a series of closely related publications designed for students who want the broadest possible introduction to technical occupations. The most effective use of these manuals is as combination textbook-laboratory guides for a full-time, post-secondary school study program that provides parallel and concurrent courses in electronics, mechanics, physics, mathematics, technical writing, and electromechanical applications.

A unique feature of the manuals in this series is the close correlation of technical laboratory study with mathematics and physics concepts. Each topic is studied by use of practical examples using modern industrial applications. The reinforcement obtained from multiple applications of the concepts has been shown to be extremely effective, especially for students with widely diverse educational backgrounds. Experience has shown that typical junior college or technical school students can make satisfactory progress in a well-coordinated program using these manuals as the primary instructional material.

School administrators will be interested in the potential of these manuals to support a common first-year core of studies for two-year programs in such fields as: instrumentation, automation, mechanical design, or quality assurance. This form of *technical core* program has the advantage of reducing instructional costs without the corresponding decrease in holding power so frequently found in general core programs.

This manual, along with the others in the series, is the result of six years of research and development by the *Technical Education Research Centers, Inc.*, (TERC), a national nonprofit, public service corporation with headquarters in Cambridge, Massachusetts. It has undergone a number of revisions as a direct result of experience gained with students in technical schools and community colleges throughout the country.

Maurice W. Roney

The Electromechanical Series

TERC is engaged in an on-going educational program in *Electromechanical Technology.* The following titles have been developed for this program:

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ELECTROMECHANISMS/FABRICATION
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MECHANISMS/MACHINES
MECHANISMS/MATERIALS

For further information regarding the EMT program or for assistance in its implementation, contact:

Technical Education Research Centers, Inc.
44 Brattle Street
Cambridge, Massachusetts 02138

Preface

The study of mechanisms is one of the oldest of the applied sciences. The early Greeks and Romans used crude pulleys and gears in a wide variety of applications; and the American industrial revolution can truly be said to have rolled on tooth gear wheels. The advent of space exploration has demanded a rebirth of interest in mechanics and mechanisms. In the past we have thought primarily of applications in the automotive, machine tool, and watchmaking fields. Today, it is more common to think of aerospace, defensive weaponry computer equipment, and precision instrument applications. These changes in emphasis have created subtle but important new demands upon training programs in mechanisms.

This material presents the topic of modern machines. It combines the elements of mechanical theory, drafting skills and practical applications. The topics treated include: graphical analysis of machines in the areas of velocity and acceleration polygons, and the dynamics of cams, gears and intermittent motion mechanisms. The materials are intended for use by technology students who have had little or no previous exposure to practical applied mechanics. Consequently, no attempt has been made to cover the material in the fine detail that would be appropriate for the experienced specialist in mechanical drives.

An attempt *has* been made to expose the student to the practical skills of graphical analysis and to the dynamic principles of operation of a variety of mechanisms.

The sequence of presentation chosen is by no means inflexible. It is expected that individual instructors may choose to use the materials in other than the given sequence.

The particular topics chosen for inclusion in this volume were selected primarily for convenience and economy of materials. Some instructors may wish to omit some of the experiments or to supplement some to better meet their local needs.

The materials are presented in an action-oriented format combining many of the features normally found in a textbook with those usually associated with a laboratory manual. Each experiment contains:

1. An INTRODUCTION which identifies the topic to be examined and often includes a rationale for doing the exercise.
2. A DISCUSSION which presents the background, theory, or techniques needed to carry out the exercise.

3. A MATERIALS list which identified all of the items needed in the laboratory experiment. (Items usually supplied by the student such as a pencil and paper are not included in the lists.)
4. A PROCEDURE which presents step-by-step instructions for performing the experiment. In most instances the measurements are done before calculations so that all of the students can at least finish making the measurements before the laboratory period ends.
5. An ANALYSIS GUIDE which offers suggestions as to how the student might approach interpretation of the data in order to draw conclusions form it.
6. PROBLEMS are included for the purpose of reviewing and reinforcing the points covered in the exercise. The problems may be of the numerical solution type or simply questions about the exercise.

Students should be encouraged to study the text material, perform the experiment, work the review problems, and submit a technical report on each topic. Following this pattern, the student can acquire an understanding of, and skill with, machines that will be very valuable on the job. For best results, these students should have completed a second course in technical mathematics (elementary applied calculus).

This material on MECHANISMS/MACHINES comprises one of a series of volumes prepared for technical students by the TERC EMT staff at Oklahoma State University, under the direction of D. S. Phillips and R. W. Tinnell. The principal author of these materials was Larry Teel.

An INSTRUCTOR'S DATA GUIDE is available for use with this volume. Mr. Larry Teel was responsible for testing the materials and compiling the instructor's data book for them. Other members of the TERC staff made valuable contributions in the form of criticisms, corrections and suggestions.

It is sincerely hoped that this volume as well as the other volumes in the series, the instructor's data books, and the other supplementary materials will make the study of technology interesting and rewarding for both students and teachers.

THE TERC EMT STAFF

Contents

The author and editorial staff at Delmar Publishers are interested in continually improving the quality of this instructional material. The reader is invited to submit constructive criticism and questions. Responses will be reviewed jointly by the author and source editor. Send comments to:

Editor-in-Chief
Box 5087
Albany, New York 12205

experiment 1 BASIC GRAPHIC ANALYSIS

INTRODUCTION. The major requirement in a graphical approach to mechanism problems is the ability to use basic drafting tools. In this experiment we will practice some of the basic drafting operations.

DISCUSSION. From the time man first made drawings on cave walls, he has been trying to develop a more effective means of nonverbal communication. One of the ways he has approached the problem has been to develop forms of writing. For normal communication, this has proved to be very effective.

However, writing has a major drawback in that it does not provide an accurate idea of certain subjects. For example, it is nearly impossible to convey a clear impression of what a complex piece of machinery looks like. This is an area for which another form of graphic representation has been developed.

The quickest way to convey an accurate description of a three-dimensional object is with pictorial or graphic methods. For simply giving an immediate grasp of what the mechanism looks like, pictorial representations are the best.

A more detailed study of the object is best accomplished with other graphic methods. Both two and three-dimensional views can be easily produced and dimensions shown.

In accordance with this line of reasoning, we will examine some of the basic components of these graphic representations or drawings.

Drawings are composed of lines representing the size, shape and surface features of an object, and are often drawn to some known scale; larger, smaller, or identical to the dimensions of the object itself.

The most important factor in making accurate drawings is the quality of the equipment. For thin, legible lines on a drawing, a 3H pencil lead is usually most suitable. The lines need to be thin in order to prevent overlapping them or misplacing a point of connection of two lines, and to enable us to measure from a line more accurately.

To draw horizontal straight lines, a T square is frequently used, and a triangle may be placed against the edge of the T square to draw vertical lines. By using two triangles, one, a 30°-60°-90°, the other, a 45°-45°-90°, with a T square as a base, lines at 15° intervals may be drawn. This procedure is illustrated in figure 1-1.

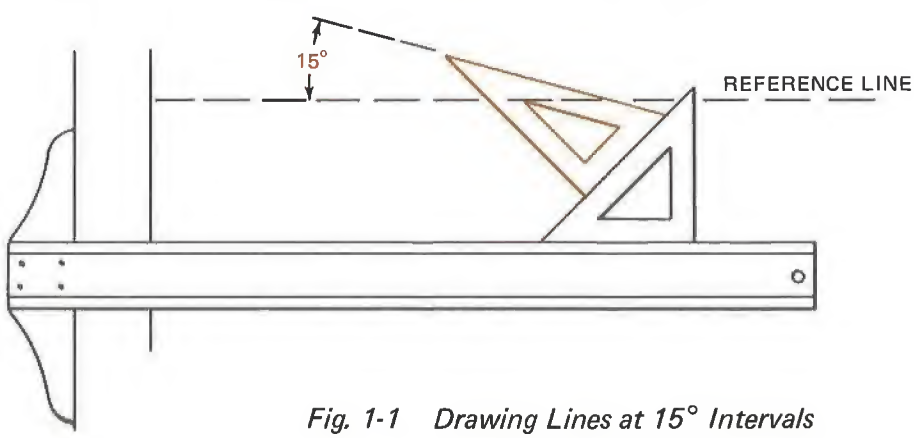


Fig. 1-1 Drawing Lines at 15° Intervals

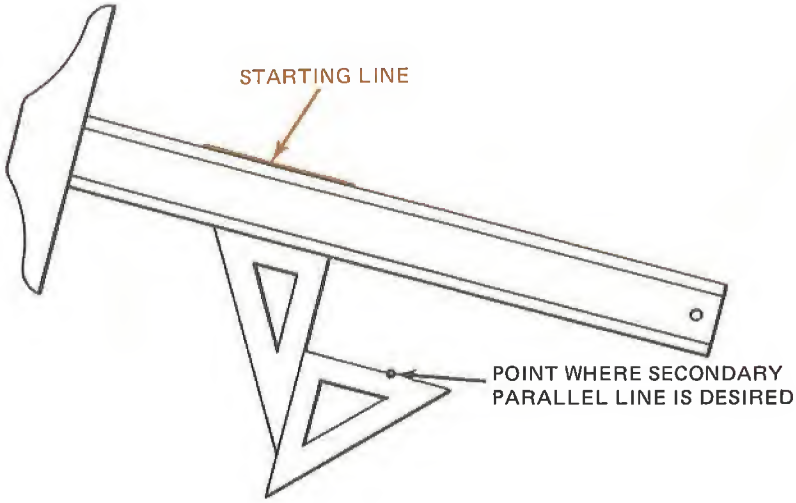


Fig. 1-2 Drawing Parallel Lines

The two triangles can be manipulated to obtain any of the other 15° increments.

In order to draw lines that are perpendicular or parallel to each other, the T square and triangle are again used. If a line is desired that is parallel to a horizontal line, the T square is used. It is slid up or down to the position desired and the new line is drawn. A line that is parallel to a vertical line is drawn by placing the right angle side of a triangle against the T square, then adjusting the T square and triangle to the position desired. Lines parallel or perpendicular to ones which are not horizontal, vertical, or in 15° increments from the horizontal, require slightly more care in drawing.

By using a little ingenuity and care in moving the instruments, lines of reasonable accuracy may be drawn. The first step in this procedure is to turn the T square over and place it parallel to the starting line. The triangles are then used in the desired configuration to draw lines either parallel or perpendicular to the original line. This method is illustrated in figure 1-2 for a secondary parallel line. With suitable variations on the above procedure, lines can be drawn either parallel with, perpendicular to, or at 15° increments from, the starting line.

For measuring angles, which are often arbitrary, a protractor is necessary. If we use a good quality protractor, and exercise reasonable care, the measurements will be accurate enough for most purposes. If a good-quality drafting machine is used, the above operations become extremely simple.

To measure the lengths of lines, some type of scale measurement is needed. An excellent type of scale to use for mechanism drawings is the *Engineer's Scale,* containing 6 scales dividing the inch into graduations which are multiples of 10. It has 10, 20, 30, 40, 50, and 60 divisions per inch. To obtain best results with this scale, all data should be converted to decimals before laying out a length. An illustration of this is the 30 scale, which is used for drawing to scales such as 1 inch = 3 inches, 30 inches, 300 inches, etc.

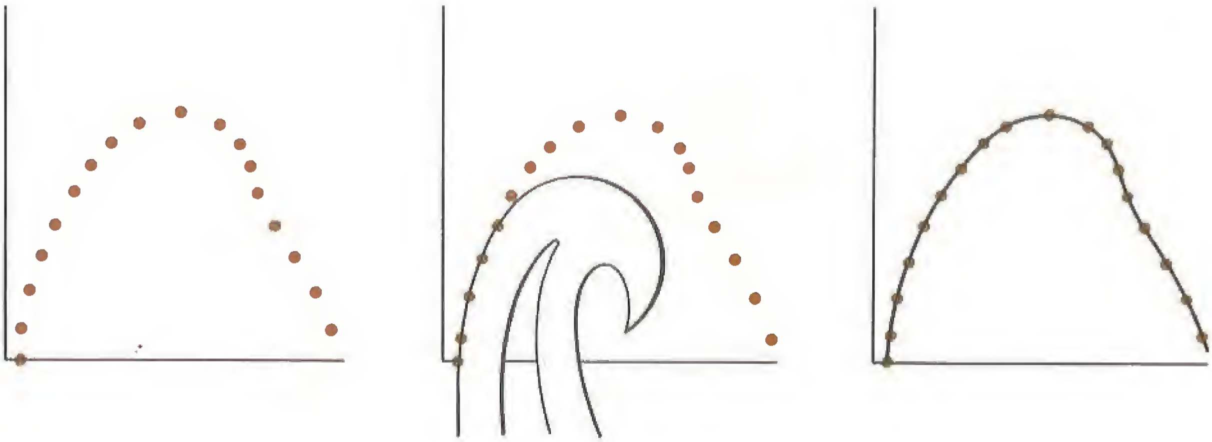


Fig. 1-3 Use of French Curve

For precise work, measurements are transferred from the scale to the drawing with dividers. Dividers essentially consist of a precision-made compass with two needle points instead of the usual lead marking tip and single needle point. Dividers can be rapidly adjusted to the desired measurement by placing one point on the zero line of the scale, then adjusting the other point to the desired displacement.

A compass is commonly used to draw circles or arcs. The radius is set using the method described for the dividers. As in using a pencil to draw lines, the compass lead must have a sharp point at all times, so that the lines will not be so wide that they cause an unreasonable loss of accuracy. The center of the circle or arc to be drawn should first be marked with a needle point. Under most circumstances a pencil point should not be used to mark the center of curvature, as this will allow the compass needle point to move, resulting in low accuracy.

Irregular, or French, curves are used to draw curves of varying radii. This type of curve often results when graphing the response of an actual system. To draw a curve of this type, the known points are located, then a line is very lightly drawn which connects each of the points. The next step is to place the French curve at the beginning of the graph, and choose a portion of it which corresponds to the curve of the graph, as closely as possible, for some distance. Then, a line is drawn along the edge of the French curve, stopping slightly short of the last points of common tangency. The French curve is then shifted to the next section of the graph, and a portion again selected which matches that of the graph, and is a continuation of the line segment just drawn. The procedure is repeated until the graph is completed, as illustrated in figure 1-3.

MATERIALS

- 1 Drafting table or board
- 1 T square
- 1 Triangle (30°-60°-90°)
- 1 Triangle (45°-45°-90°)
- 1 Compass
- 1 Divider
- 1 Engineer's scale
- 1 Protractor
- 1 Irregular curve
- 6 Drafting paper, approx. 8-1/2 in. X 10-1/2 in.

PROCEDURE

1. Inspect each of your instruments to be sure that they are undamaged.
2. Using the appropriate equipment, draw a rectangle 3.85 in. wide and 2.37 in. high, in the upper half of a page of drafting paper.
3. Using the methods described in the discussion, draw a circle of radius 2.09 in. in the lower half of the page. Then, inscribe diameter lines in it, at increments of 15° from the horizontal, for the full 360° of the circle.
4. Draw a horizontal line approximately 5 to 6 in. long in the upper portion of another page.
5. Next, draw a line which is perpendicular to the original line in the lower left part of the page.
6. Now draw a line which is parallel to the original line in the lower right portion of the page.
7. Draw a line approximately 5 or 6 in. long, inclined at an angle of 37° clockwise from the horizontal, in the upper portion of another page.
8. Repeat steps 5 and 6, using the line just drawn as the starting line.
9. Draw a parallelogram with $\alpha = 67.5°$, $\ell_1 = 2$ in., and $\ell_2 = 4$ in., where α, ℓ_1, and ℓ_2 are as shown in figure 1-4.
10. Using the irregular curve, draw a *smooth* curve through the points in figure 1-5.

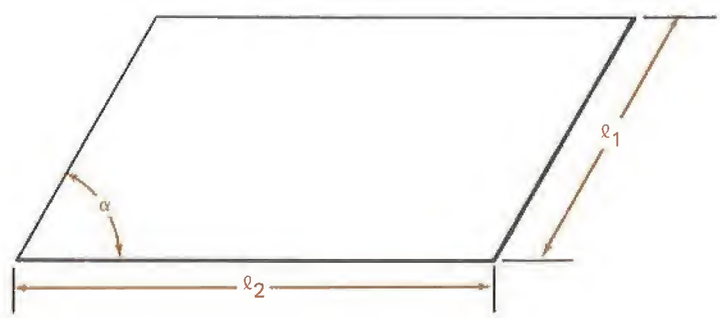


Fig. 1-4 Parallelogram Measurement

ANALYSIS GUIDE. Explain, in your own words, what you think could cause the major source of error in each of the procedure steps. Consideration might be given to such things as human error in measuring, slippage of instruments, etc.

PROBLEMS

1. Describe a procedure for finding twice the variation in the 90° angle of a right triangle. (*Assume* the T square is perfectly straightedged).
2. To check the assumption that the T square is perfectly straightedged, describe a procedure that could be used for verification of this.

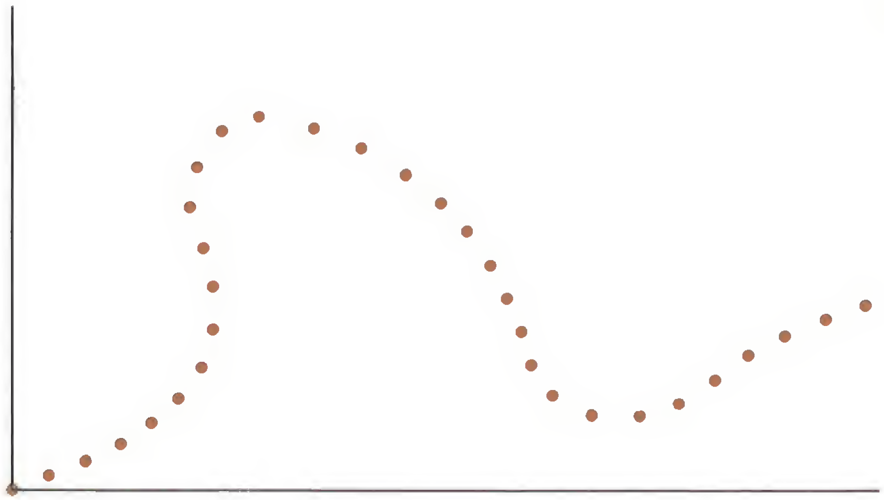


Fig. 1-5 Use of Irregular Curve

3. Draw two circles of different radii on a sheet of drafting paper, using an arbitrary distance between the two than draw the *four tangent* lines connecting them.

4. Draw two lines intersecting at right angles at the top of another sheet of drafting paper. Then, using only a compass and straightedge, draw a connecting line at a 45° angle to both of the lines. The desired result should be similar to figure 1-6.

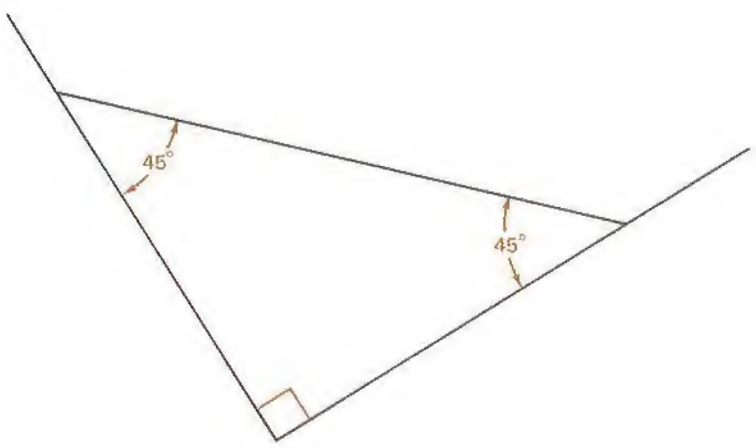


Fig. 1-6 Problem 4 Result

5. Draw two lines intersecting at right angles in the bottom part of the page each as near 5 in. long as possible. Mark off increments of 0.1 in. on each of them, for 4 inches. Using a straightedge, connect the points wtih a number of straight lines such that they approximate an arc of a circle.

experiment 2 PLOTTING EQUIPMENT CALIBRATION

INTRODUCTION. One of the difficulties encountered in laboratory work involves the calibration of experimental equipment. In this experiment, we will examine the calibration of a device that can be used to plot displacement as a function of time.

DISCUSSION. A problem inherent in all mechanical devices is that of knowing what motion individual machine parts are performing. If we are located near the part in question, a mechanical linkage can be used to provide this information. An example of this situation is the speedometer on an automobile, which is simply a mechanically-driven velocity meter.

However, when we are located at a relatively greater distance from the part, mechanical linkages often do not function too well. Frictional losses, inertial effects, etc., influence the output information to too large an extent. This is a situation for which mechanical-electrical transducers are extremely well adapted, with the output information transmitted electrically to the indicating device.

This situation is illustrated by the RPM indicators for the engines of an ocean liner. Here the observer may be several hundred yards from the engine. Another example is the RPM indicator for an electrical generator in a power substation, where the observer may be hundreds of miles from the generator.

Such mechanical-electrical transducers often have some important advantages over purely mechanical linkages. They make remote readouts of information possible, lessen friction and inertia effects, and allow amplification of output information.

A mechanical-electrical transducer commonly used is the variable-resistance transducer. The governing equation for this device is that of the effective resistance for a linear electrical conduction device:

$$\boxed{R_e = \frac{\rho L_e}{A}} \tag{2.1}$$

where

R_e = effective resistance, ohms

ρ = material resistivity, ohms/ft

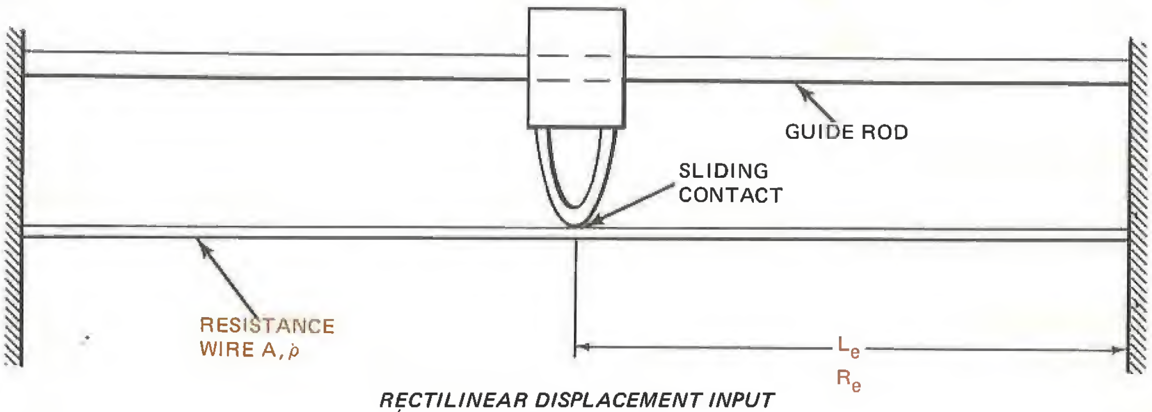
L_e = effective conductor length, ft

A = conductor cross-sectional area, ft^2

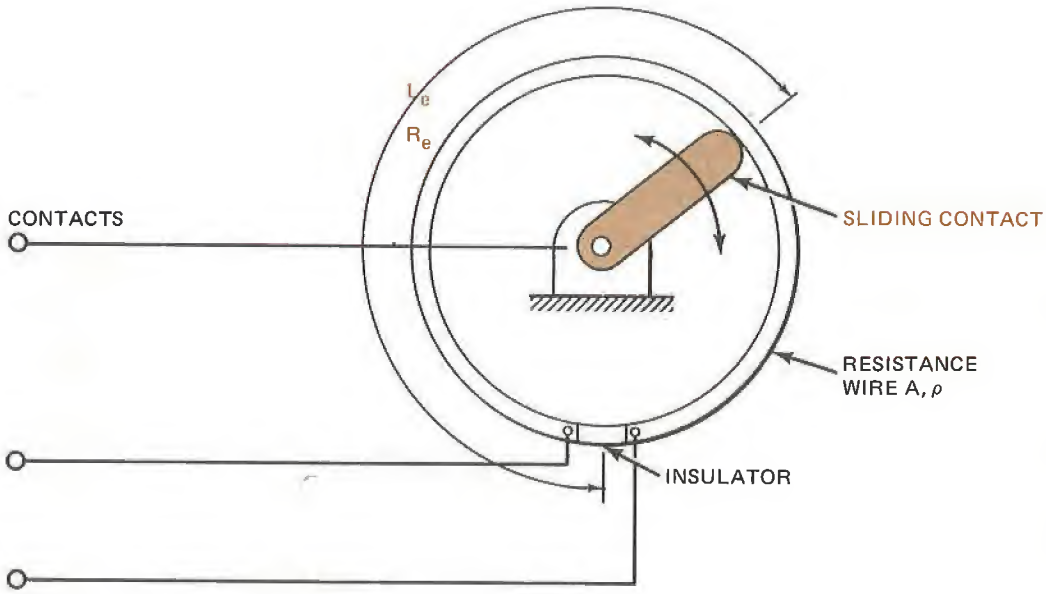
From equation 2.1, we can see that the effective resistance can be varied by changing the effective conductor length

One transducer which utilizes this characteristic is the sliding-contact potentiometer. This device changes a mechanical displacement input into a resistance output by varying the effective conductor length. Two types of this transducer encountered in practice are the rectilinear displacement input and the angular displacement input. Illustrations of these mechanisms are shown in figure 2-1.

Variable-resistance transducers encountered in industrial applications ordinarily use a resistance element which is composed of an insulating core wound with many turns of resistance wire, either spaced or insulated so that there is no shorting in the system. This arrangement is shown in figure 2-2.



RECTILINEAR DISPLACEMENT INPUT



ANGULAR DISPLACEMENT INPUT

Fig. 2-1 Potentiometer Types

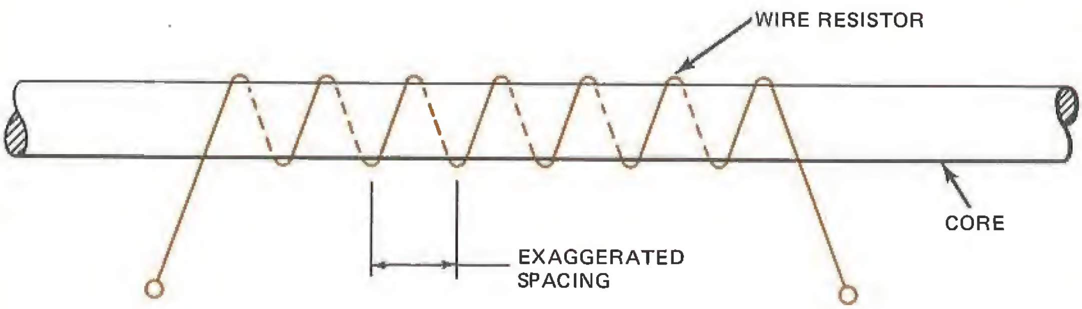


Fig. 2-2 A Wire-Wound Potentiometer

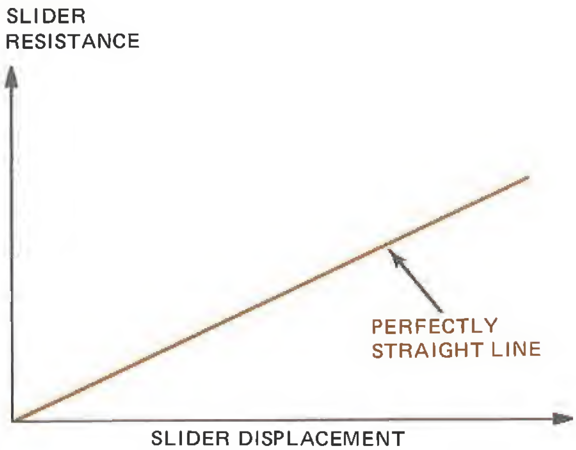


Fig. 2-3 Ideal Characteristic Curve

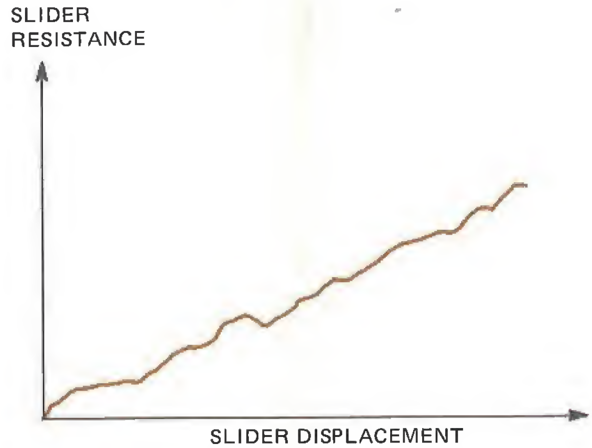


Fig. 2-4 Actual Characteristic Curve

With this arrangement, the effective length of the wire resistor can be increased, although the actual length seen by a slider remains the same as that of a long, straight wire resistor.

Transducers with wound-wire resistors are often more useful than a straight wire resistor because there is a greater change in resistance per unit length if the wires of both resistances have the same resistivity.

This introduces a problem which technicians have encountered since men like Galileo and Leonardo da Vinci first attempted to formulate mathematical equations describing the behavior of actual systems. The problem arises because *exact* mathematical equations cannot be written and solved for a system, unless it is an extremely simple one. However, by making appropriate simplifying *assumptions* about the system, the complicated exact equation can often be reduced to an approximate equation which may be easily solved.

In equation 2.1, there are many assumptions which are not readily apparent. Among these are assumptions that the area and resistivity of the wire resistor are constant. This implies that there are no voids in the resistor material and that the resistor has exactly the same composition at all points along its length. It is also implied that there is perfect contact between the slider and the resistor at all times, and that the resistivity of the wire is constant. This last assumption is the most critical, as it incorporates the majority of the other assumptions. If the resistor material were perfectly linear, it would have the characteristic resistance-displacement curve shown in figure 2-3.

Because no actual resistor is absolutely linear, it is likely that the transducer will have a resistance-displacement curve similar to that shown in figure 2-4.

From figure 2-4, we see that there is not actually a simple mathematical relationship between the resistance and displacement of the slider. Along with this, there is the fact that the actual characteristic curve is constantly changing, as the resistor heats up or cools down, causing the molecular condition of the resistor material to change. Therefore, the actual characteristic curve is not only of a shape which is virtually undefinable, but the curve is also changing with respect to time.

The curve can be approximated by a straight line and be well within reasonable accuracy limits. Applying this procedure to figure 2-4 then, the technician linearizes the curve by assuming a constant resistivity for the resistor material.

With this assumption (and other either stated or implied), an equation such as equation 2.1 can be developed, and readily solved.

MATERIALS

- 1 Strip chart or X-Y recorder
- 1 DC power supply (0–40V)
- 1 Rectilinear potentiometer 5kΩ
- 1 Engineer's scale
- 1 Divider

PROCEDURE

1. Inspect all of your components to be sure they are undamaged.
2. Check the maximum rated voltage and current of the potentiometer. If the DC power supply voltage is above the rating of the potentiometer, it may be necessary to include a series resistor in the circuit to limit the current to a safe level. This is shown in figure 2-5.
3. Connect the three components as shown in figure 2-6 with the slider connected across the Y-axis of the recorder. If necessary, include the limiting resistor illustrated in figure 2-5.

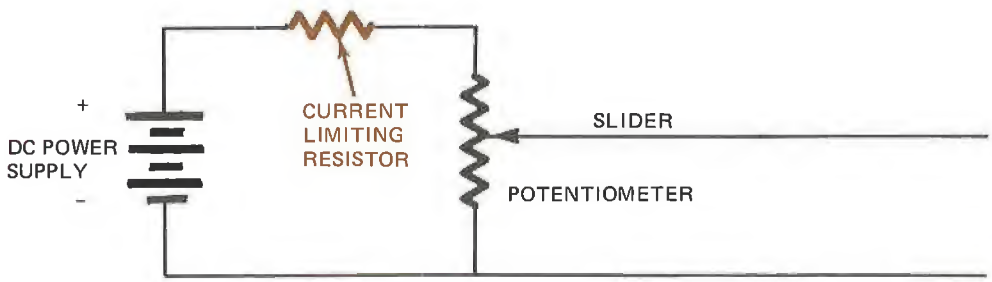


Fig. 2-5 Circuit with Current Limiting Resistor

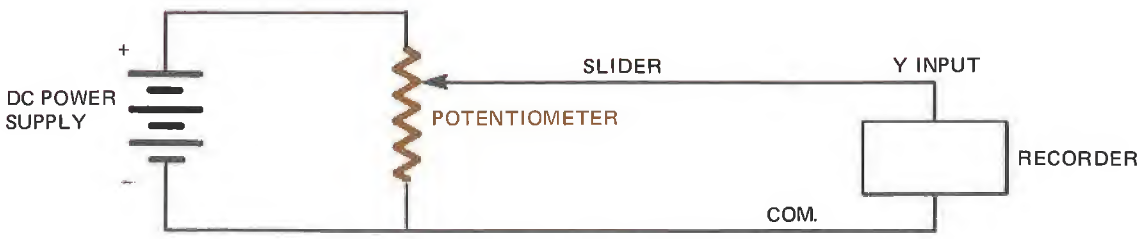


Fig. 2-6 The Experimental Circuit

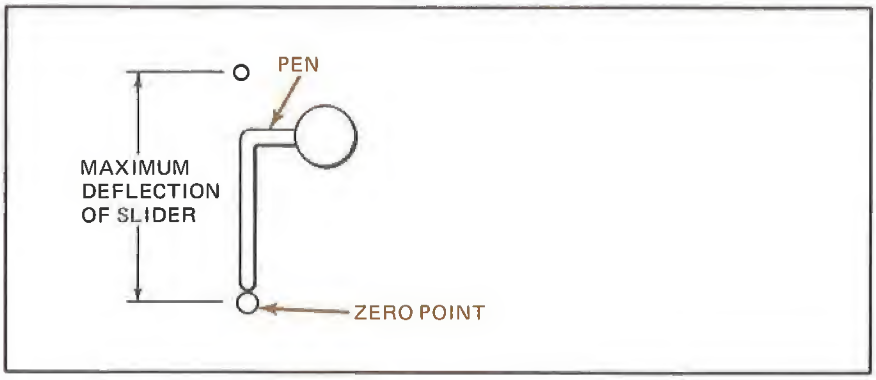


Fig. 2-7 Results of Steps 4 and 7

4. Using an engineer's scale, measure the total travel possible for the potentiometer slider. Transfer this measurement to the recorder paper, using a convenient zero point if one is not printed on the paper. We will use this measurement to calibrate the system.

5. Turn on the recorder *first*, then turn on the DC power supply. Carefully increase the voltage while simultaneously adjusting the Y-axis zero control to prevent full-scale deflection and possible damage to the recorder.

6. Pull the slider out to its maximum displacement, again using the Y-axis zero control to prevent full-scale defelction on the recorder.

7. Use the Y-axis and X-axis zero controls to place the pen directly over the zero point marked in step 4 for an X-Y recorder. For a strip-chart recorder, it will be necessary to use the chart drive to obtain movement on the X-axis. The results of this initial setup should be as shown in figure 2-7.

8. Using a combination of the DC power supply voltage level control, the recorder Y-axis sensitivity control, and the recorder Y-axis zero control, adjust the system so that when the potentiometer slider is pushed in and out for maximum displacement, the pen will move the corresponding displacement on the recorder between the points marked in step 4. **NOTE: If the pen moves down instead of up when the slider is pushed in, reverse the connections of the DC power supply.**

9. With the slider all the way out, mark the X-axis for about eight inches by putting the pen down and sweeping it across the paper. Repeat this procedure with the slider all the way in.

10. Measure the distance between the two lines to verify that it is the same as the maximum displacement of the slider.

11. To check the linearity of the potentiometer, move the slider in 0.5-inch increments from zero to maximum displacement and mark a line parallel to the X-axis for each increment as in step 9. Measure the displacement between each line on the paper to see whether it is 0.5 inch displacement. Record the measurement.

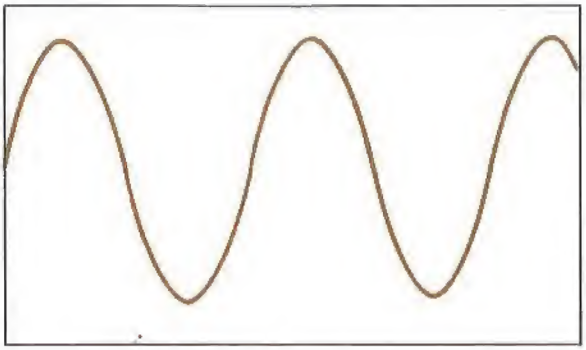


Fig. 2-8 Sinusoidal Wave Output

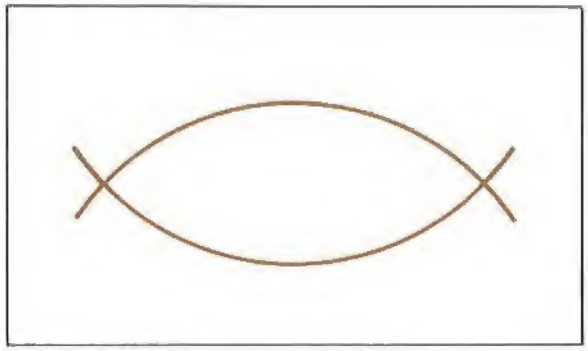


Fig. 2-9 Two Parabolas

12. Compute the percent error for each increment using

$$\frac{0.5 - \text{pen displacement}}{0.5} \times 100$$

and record the results.

13. With the X-axis control set on a time base or sweep function, try to generate a sinusoidol wave by moving the slider in and out. The desired result is illustrated in figure 2-8.

14. Using the procedure in step 13 generate two parabolas of the form shown in figure 2-9.

15. Now generate a damped sinusoid of the form shown in figure 2-10.

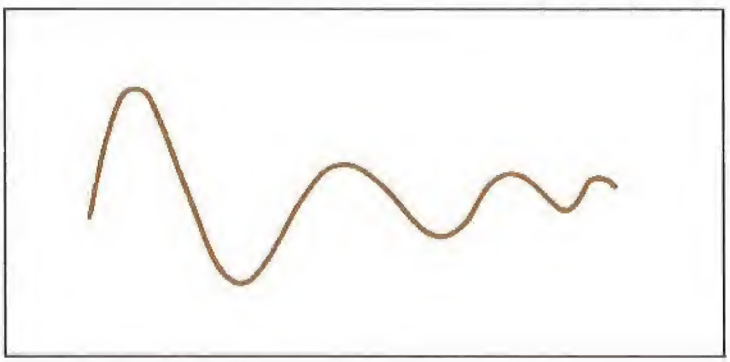


Fig. 2-10 A Damped Sinusoid

ANALYSIS GUIDE. What do you feel are the major sources of error in checking the linearity of the potentiometer? Consideration should be given to such things as human error in measurement, variation of conditions in the system, and friction in the system.

PROBLEMS

1. If the maximum displacement reading of the potentiometer is four inches with the recorder sensitivity set at 10 volts/division, what should the maximum displacement reading be when the sensitivity level is changed to five volts/division?

2. What effect will the change in problem 1 have on the slope ($\frac{YS}{Yt}$) of a line if the potentiometer is moved in exactly the same manner at both sensitivity settings?
3. If the potentiometer slider is pulled all the way out, then pushed in as quickly as possible, will the recorder pen give an accurate record of the slider movement? If not, why?
4. Describe a simple procedure to use that will give an approximate graph of a circle on the recorder.
5. If you were to use a potentiometer that was definitely nonlinear, what procedures would you use to calibrate it and obtain the actual displacement curve from the recorded curve?
6. Graph the following points by hand: S = 0, t = 0; S = 0.2, t = 1; S = 0.8, t = 2; S = 1.6, t = 2.6; S = 3, t = 3.6; S = 3.5, t = 4.8; S = 3, t = 6, with S in inches and t in seconds. When 3.1 seconds have elapsed, what is S? When S = 1.5 inches, what is t?
7. Assuming that the curve in problem 6 continues smoothly, when t = 7 seconds, what is the value of S?

experiment 3 *LINK POINT CURVES*

INTRODUCTION. A problem that frequently arises in working with mechanisms is that of plotting the characteristic curves for points on a linkage. In this exercise we will investigate this problem.

DISCUSSION. In order to design a new linkage system or modify an existing one, it is often necessary to know what motion a point (normally an end point) on a link will undergo. If an exact solution is desired, mathematical equations can be written describing the motion, and solved by computer at desired increments in the cycle. A method that quickly gives results that are less accurate than a computer solution, but normally within reasonable tolerance limits, is to use a graphic approach. This kind of approach consists of drawing the linkage at several intervals in the cycle with the number of intervals determined by the accuracy desired. An illustration of this procedure is shown in figure 3-1 for a typical four-bar mechanism.

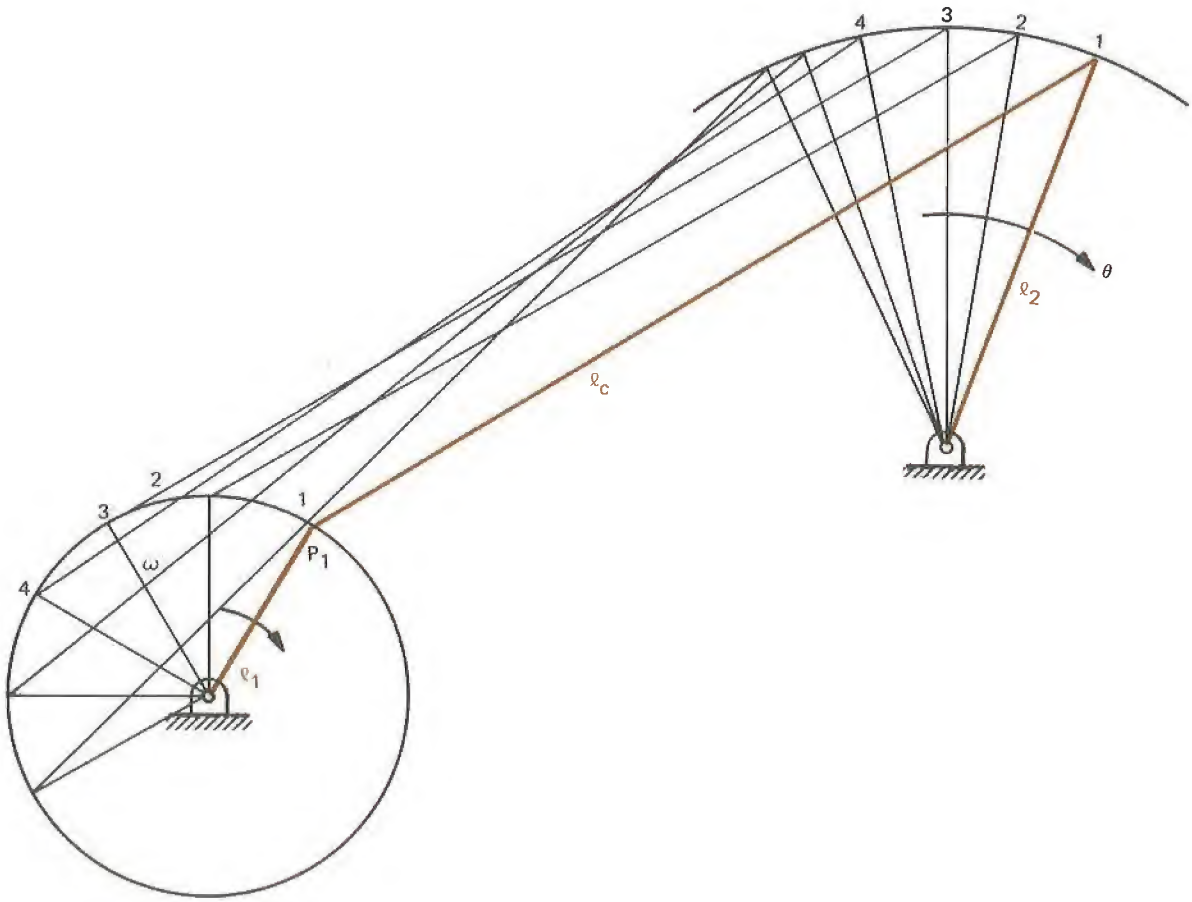


Fig. 3-1 Four-Bar Linkage

This method only works, however, for linkages which are constrained to move in a particular path. If a link is floating, as illustrated by link ℓ_{c2} in figure 3-2, there is no way the displacement of points can be described as a function of the input crank, ℓ_1, angular velocity. Links ℓ_{c1}, ℓ_{c2}, and ℓ_2 can perform any of a large number of different motions as the input crank is turned.

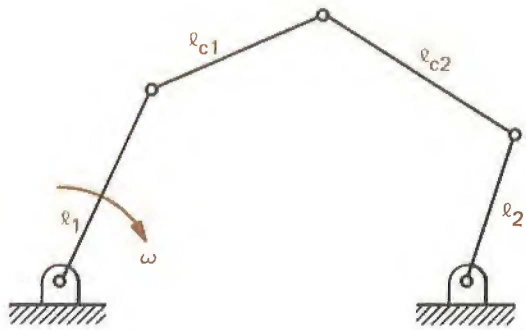


Fig. 3-2 Floating Link

If, however, the motion of all members can be described completely as a function of the input crank angle, as is the case in figure 3-3, then a time-displacement curve may be drawn for *any* point along the linkage.

In figure 3-3, if ω is known, time versus θ can be plotted, giving a time-displacement curve for P_1, since θ is proportional to the length of the circular arc, S. This relationship is shown in figure 3-4. We also observe that a displacement curve for P_2 may be plotted, since the linkage undergoes constrained motion.

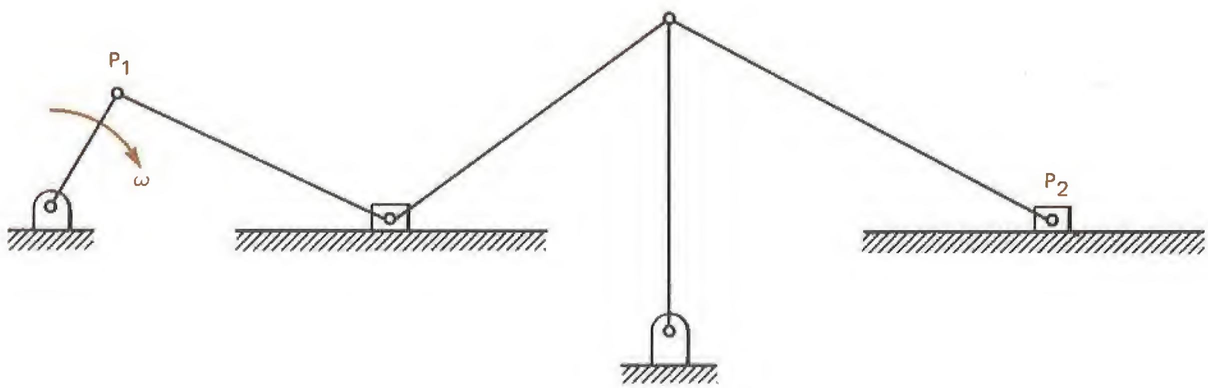


Fig. 3-3 Constrained Motion

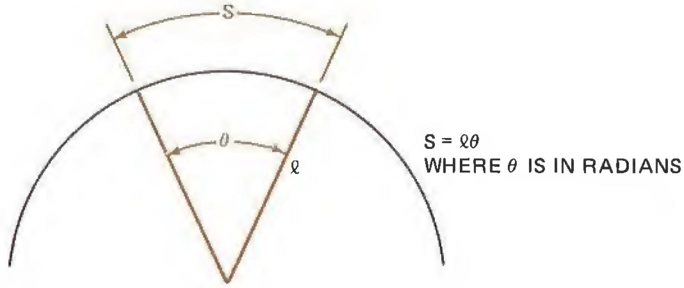


Fig. 3-4 Circular Arc

When ω is unknown, the crank angle is plotted against the follower angle, giving an angular displacement versus angular displacement curve of the same shape as the time displacement curve.

When θ is large enough, it may be measured with a protractor. If it is inconvenient to measure, or if the angle is too small to accurately measure with a protractor, a pair of calipers may be used to directly find the arc length. This will involve some error since the calipers linearize the arc length, but for large radius circles and small arc lengths the error will be very small. For a constant value of ω, the time-displacement curve for point P_2 in figure 3-1 will have the shape of figure 3-5.

Fig. 3-5 Displacement Curve

Angular velocity, ω, is the time rate of change of displacement, or the slope of the displacement curve at a point. Since it is difficult to find the slope exactly by graphic means, there are two methods that are frequently used to approximate the slope. The first is to draw a line that is tangent to the curve and assume that it has the same slope, $\Delta Y/\Delta X$, that the curve does at the point of tangency. This method is normally used to graphically differentiate a curve. The second is to make a right triangle with the hypotenuse formed by a portion of the curve and $\Delta Y/\Delta X$ as the numerical value of the slope. These procedures are shown in figure 3-6.

Drawing a tangent line has the advantage that it will give good results even though the curve changes very sharply. However, for curves that do not change abruptly, forming the right triangle also works reasonably well.

Acceleration is obtained from the velocity curve in the same way that the velocity is derived from the displacement curve. This results in the second time-derivative of the displacement curve or the first derivative of the velocity curve.

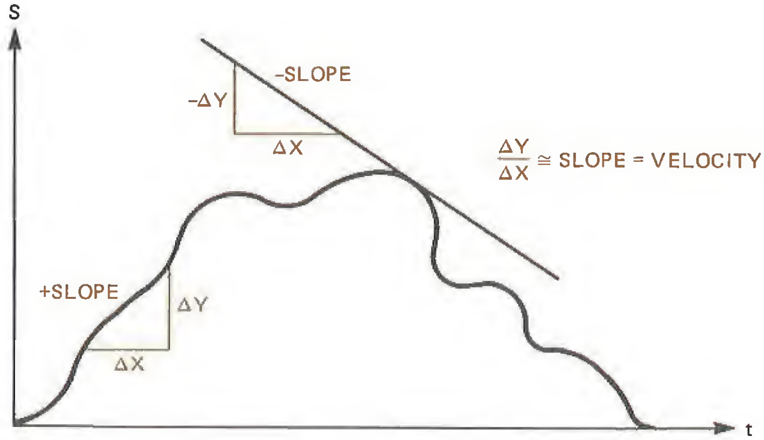


Fig. 3-6 Slope of a Line

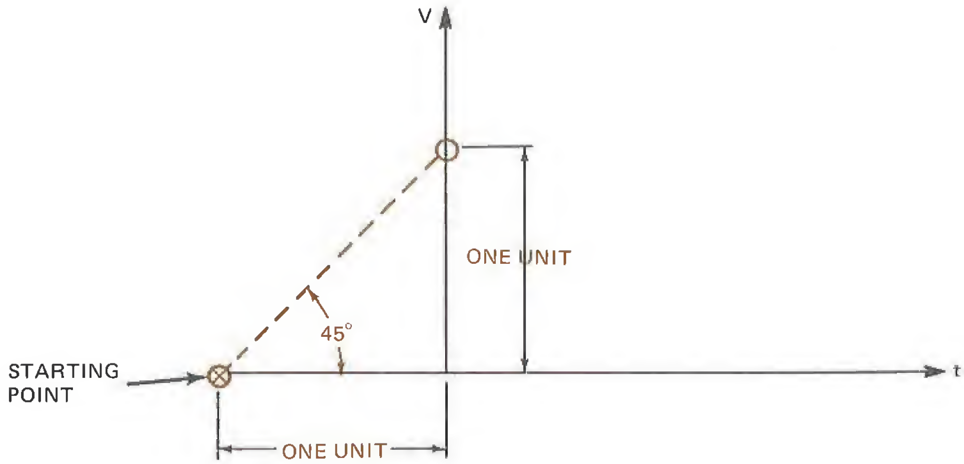


Fig. 3-7 Point Placement

Velocity and acceleration curves can be drawn by finding the numerical value of the slope of the displacement and velocity and then plotting each value. For complex curves this is a very tedious method. Graphic differentiation, however, provides a quick and reasonably accurate way to get the velocity and acceleration curves.

Graphic differentiation consists of drawing the tangent lines to the starting curve and transferring the slope to a second curve at the same time value without computing the numerical value of the slope. The procedure is to mark a point to the left of the origin of the second curve. If the point is placed one unit to the left of the origin, then one unit up on the velocity axis has a value of unity, as illustrated in figure 3-7.

The displacement curve is differentiated into a velocity curve by several steps. Tangent lines are drawn to the displacement curve at intervals, with a larger number of tangents where there is an abrupt change of curvature, for better accuracy. Then a straightedge is placed parallel to the tangent and slid down to the starting point on the velocity curve. The best way to do this is to slide it along the edge of a T square. A line is lightly marked on the velocity axis. Then a horizontal line is drawn across to the corresponding time and the point marked. This is repeated until the complete velocity curve may be drawn by curve-fitting between the marked points. The acceleration curve is obtained from the velocity curve in the same manner. Figure 3-8 shows a complete set of curves for a typical displacement plot.

This process can be applied to the four-bar linkage displacement curve of figure 3-3 and the velocity and acceleration at all points in the cycle found.

The process of differentiation may also be reversed, and the acceleration curve *integrated* into the velocity curve, which in turn is integrated into the displacement curve. Integration may be accomplished by marking several lines of the correct slope on the velocity plot, at corresponding time intervals. When the lines are marked at all the desired times, a smooth curve is drawn through the slope lines.

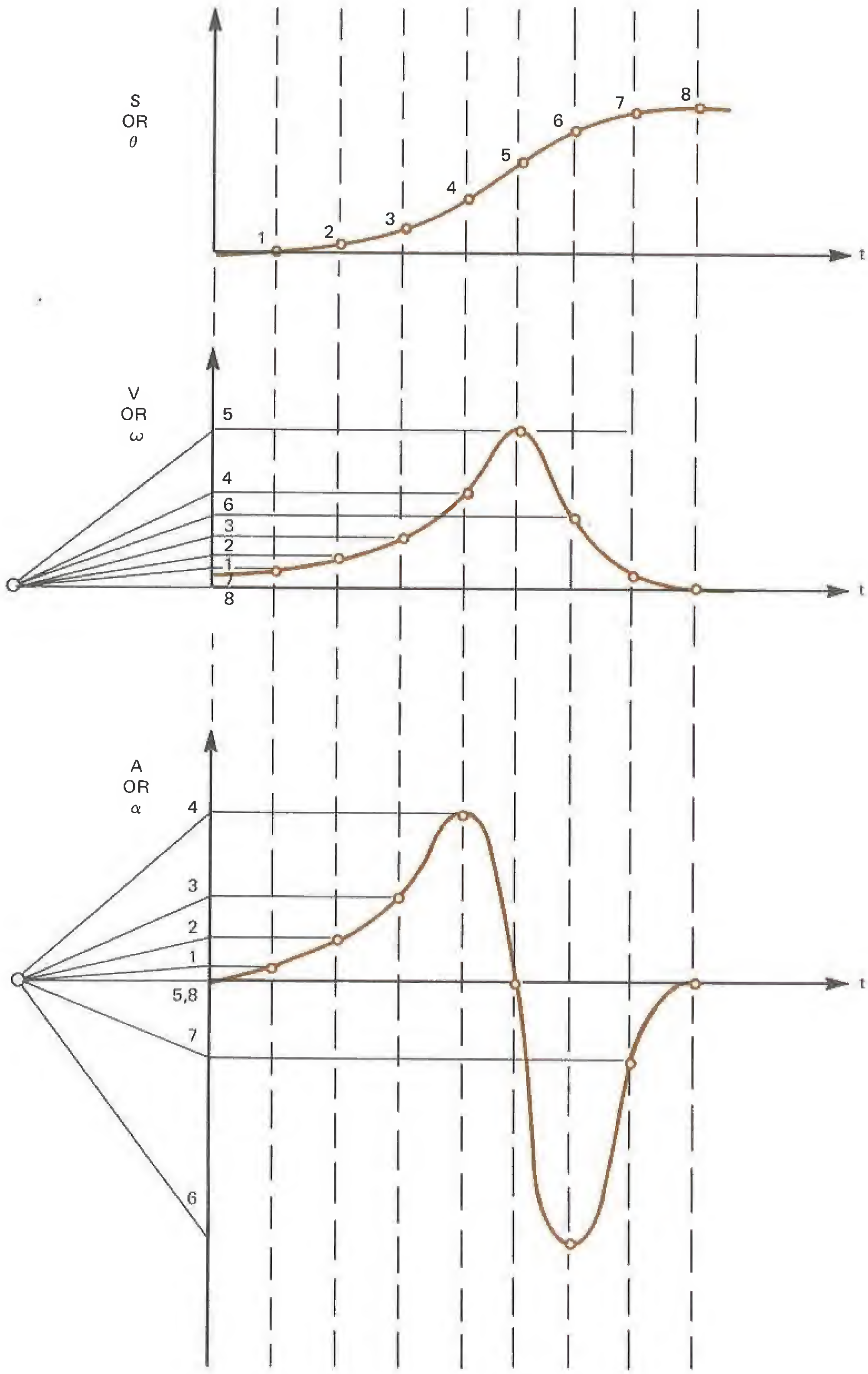


Fig. 3-8 Graphic Differentiation

The integration of the velocity curve is extremely useful for such things as assembly line work and machining operations. For instance, it may be necessary to develop a linkage to place a cover on a bar of soap on an assembly line that is moving at a particular speed. In this instance the velocity of the assembly line is known, and the displacement curve for the linkage is desired. Therefore, the velocity curve is integrated into the displacement curve and a mechanism built which will generate the desired displacement function for the necessary time. The acceleration curve should also be found in order to prevent undesirably high accelerations, which could cause excessive wear on the linkage.

When drawing a displacement curve, the limiting positions of the linkage should be plotted because they occur where the peak displacements do. This approach is illustrated in figure 3-9 for a slider-crank linkage.

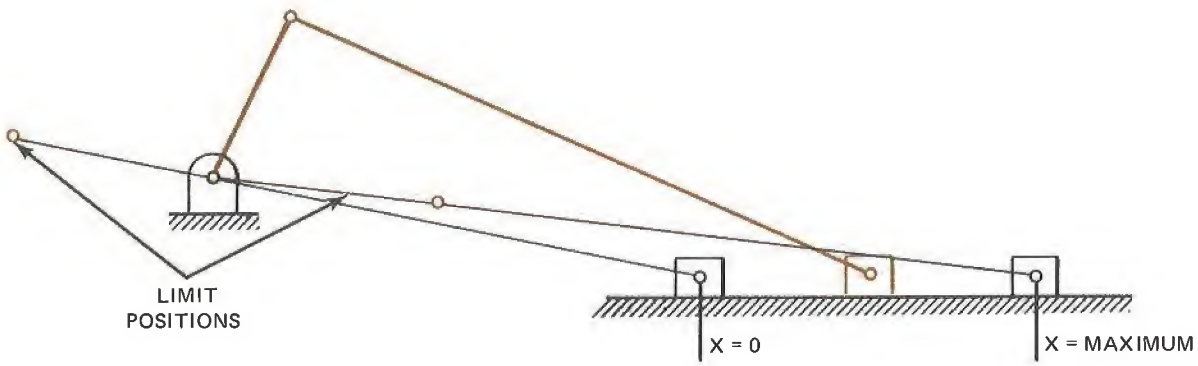


Fig. 3-9 Limiting Positions

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Shaft hangers with bearings
2 Bearing holders with bearings
2 Shafts 4 x 1/4
2 Collars
1 Shim
1 Lever arm 1 in. long with 1/4 in. bore hub
2 Clamps
1 Lever arm 4 in. long with 1/4 in. bore hub
2 Straight links 6 in. long
1 2-3 in. gear
2 Sheets of linear graph paper
3 Sheets of drafting paper
1 Divider
1 Straightedge
1 Engineer's scale
1 Drafting pencil
1 Rectilinear potentiometer
1 Rotary potentiometer
1 DC power supply
1 X-Y or strip-chart recorder

PROCEDURE

1. Check all of your instruments and components to be sure they are undamaged.
2. Assemble the linkage and instruments as shown in figure 3-10.

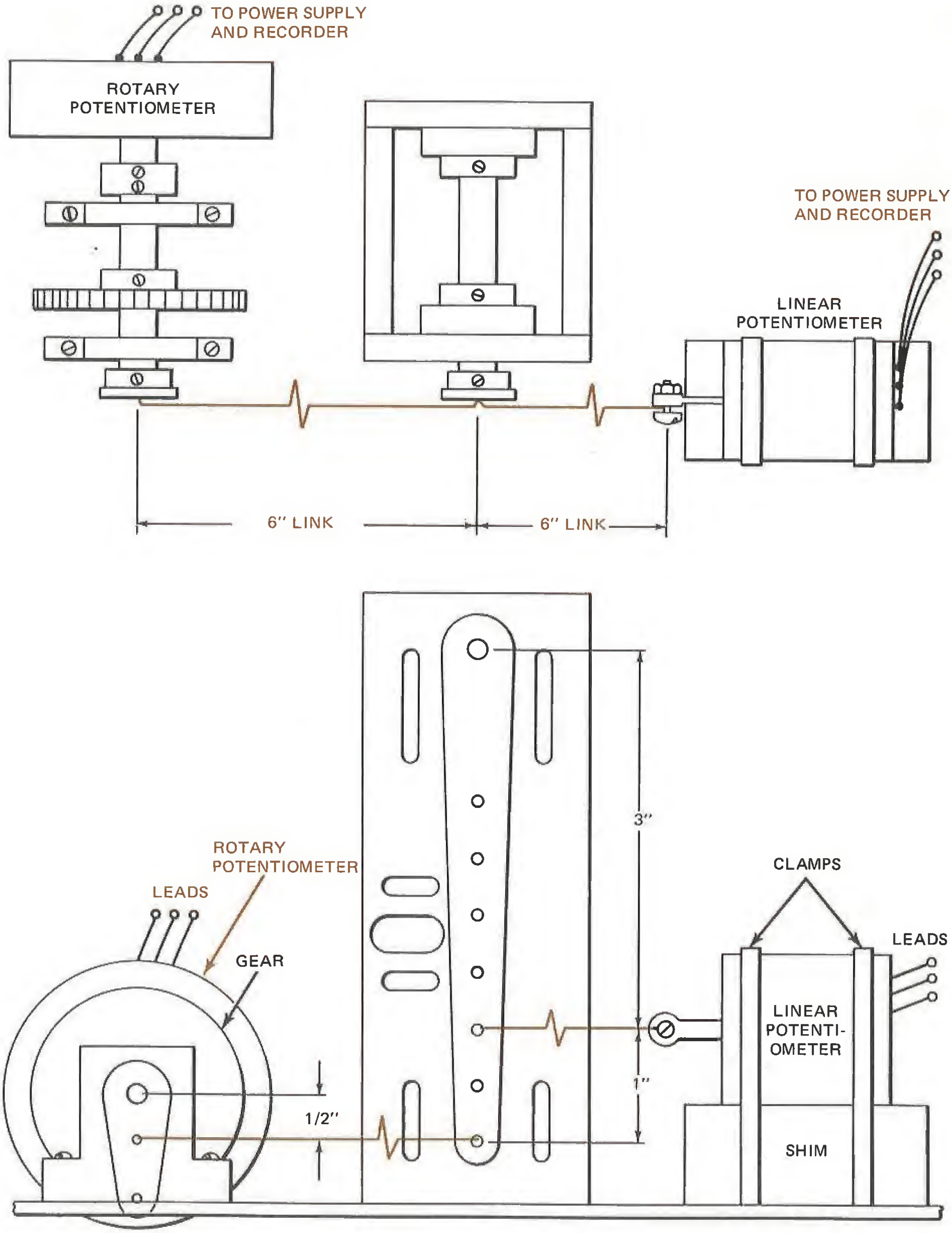


Fig. 3-10 The Experimental Setup

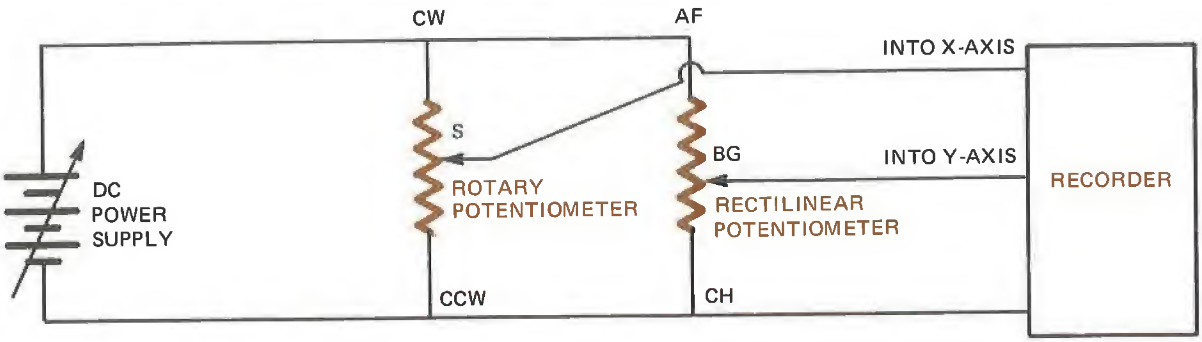


Fig. 3-10 The Experimental Setup (Cont'd)

3. Check to be sure that the *rotary* potentiometer is connected across the X-axis to provide sweep action, and that the *rectilinear* potentiometer is connected across the Y-axis to give the displacement function.

4. Turn the recorder on first, then the power supply. Adjust, by trial and error, the voltage control on the power supply and the sensitivity settings on the recorder, to obtain approximately a 7-inch maximum deflection on the X-axis and a 4-inch maximum deflection on the Y-axis. (Note: Zero the rotary and rectilinear potentiometers so that they form a single peak.)

5. Rotate the 1/2-inch input link through at least one revolution to obtain a plot of the follower displacement versus input crank displacement.

6. Graphically differentiate the displacement curve to obtain the velocity curve for the follower link at the point of connection of the rectilinear potentiometer.

7. Graphically differentiate the velocity curve to obtain the acceleration curve.

8. Try rotating the 1/2-inch drive link at various speeds to verify that the displacement plots are exactly the same so long as the velocity is not so high that the recorder pen floats.

9. Draw the linkage in figure 3-11.

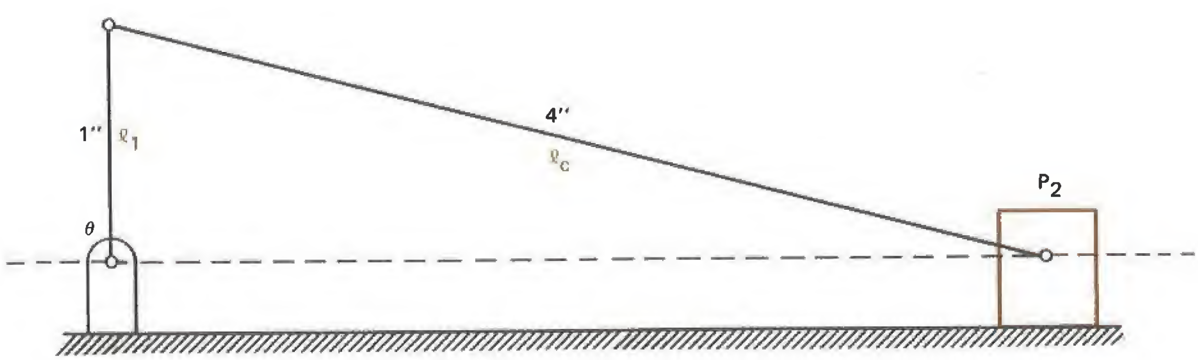


Fig. 3-11 Linkage for Step 9

10. Obtain the displacement curve for one cycle for point P_2 in figure 3-11.
11. Find the velocity curve from the displacement curve in step 10.
12. Graphically differentiate the velocity curve in step 11 to find the acceleration curve.
13. Graphically *integrate* the acceleration curve from step 12 to find the velocity plot for point P_2.
14. Graphically *integrate* the velocity curve from step 13 to find the displacement curve.
15. Plot the displacement curves from steps 10 and 14 on the same graph to the same scale and visually compare results.

ANALYSIS GUIDE. What factors do you think would cause errors in the displacement curve obtained in step 5? Discuss how the two curves from step 10 and 14 compared and the errors you think would cause differences in the curves. You might consider such things as human error in measurement, and incorrect angles caused by twisting of the straightedge in transferring slopes, etc.

PROBLEMS

1. Describe in your own terms, using diagrams if necessary, the procedure for graphically integrating a curve.
2. For the time-displacement curve shown in figure 3-12, what is the velocity at 0.5 seconds?

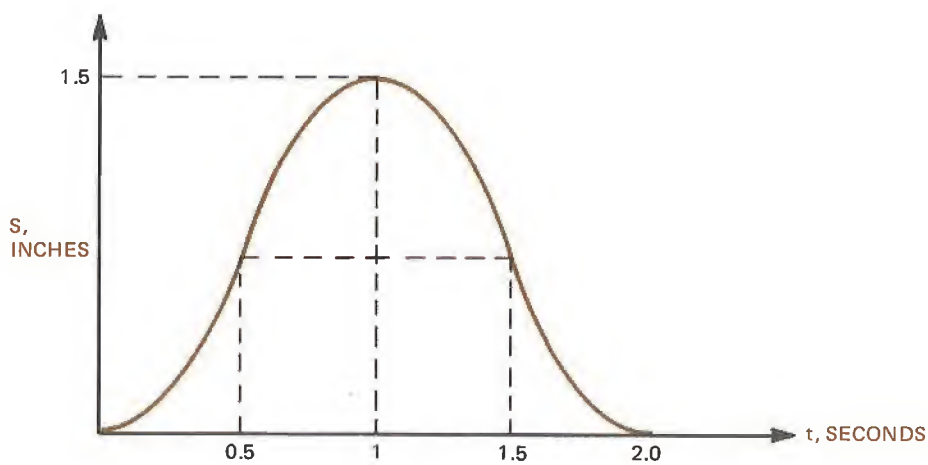


Fig. 3-12 Displacement Curve

3. From figure 3-12, what is the acceleration at 0.5 and at 1.5 seconds, assuming that the displacement curve is symmetric about t = 1.0 sec?

4. Construct a smooth graph of the following coordinates: $t = 0$, $S = 0$; $t = 0.5$, $S = 0.5$; $t = 1$, $S = 1$; $t = 1.25$, $S = 1.43$; $t = 1.5$, $S = 1.74$; $t = 1.75$, $S = 2.17$ (*peak*); $t = 2$, $S = 1.74$; $t = 2.25$, $S = 1.43$; $t = 2.5$, $S = 1$; $t = 3$, $S = 0.5$; $t = 3.5$, $S = 0$, where t is in seconds and S, in inches.
5. What is the velocity between 0 and 1 seconds for the graph constructed in step 4?
6. What is the acceleration from 0 to 1 second and from 2.5 to 3.5 seconds for the graph in problem 4?
7. What is the velocity and acceleration at 1.75 seconds for the graph in problem 4?
8. Why should the displacement plots on the recorder have the same curve for a wide variety of angular velocities in this exercise?
9. Would the displacement plots have the same curve if the rotary potentiometer is disconnected and the X-axis set on a time-sweep function?

experiment 4 *INSTANTANEOUS CENTERS*

INTRODUCTION. The motions of mechanisms such as pulleys, belts, rollers, and cams are relatively easy to predict. Elements of a mechanism in *combined* rotation and translation often involve more complicated methods of analysis. In this experiment we will investigate one of these methods; that of instantaneous centers.

DISCUSSION. Consider a link which is in combined rotation and translation, such as link ℓ_c in figure 4-1. From the two positions shown, it is obviously difficult to write a simple mathematical expression for the movement of points along link ℓ_c.

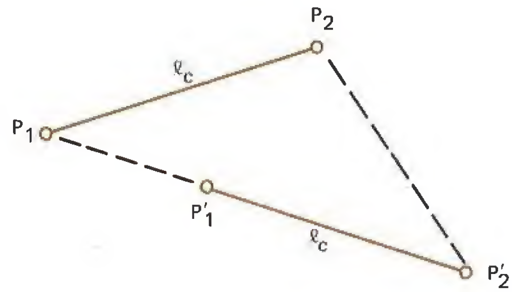


Fig. 4-2 Link Movement

Examining only the displacement of link ℓ_c, as in figure 4-2, the dashed lines represents the movement of the end points of link ℓ_c.

By placing a perpendicular bisector on lines P_1 - P'_1 and P_2 - P'_2 as illustrated in figure 4-3, the point of intersection, Q, may be considered as a *center of rotation,* or *centro,* for an infinitesimal displacement of link ℓ_c.

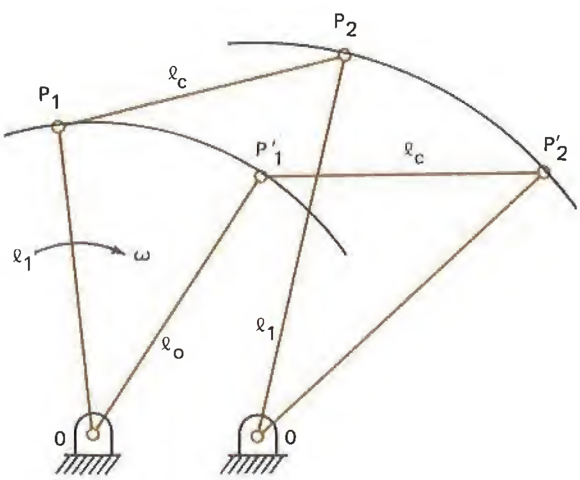


Fig. 4-1 Combined Rotation and Translation

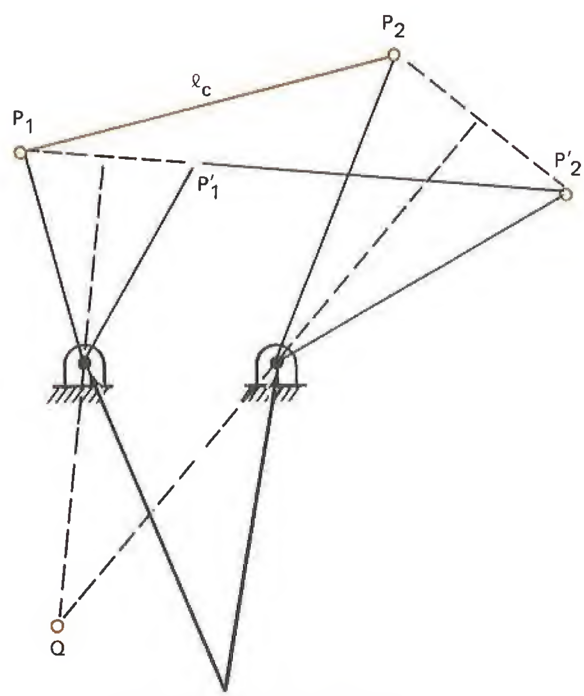


Fig. 4-3 Point of Intersection

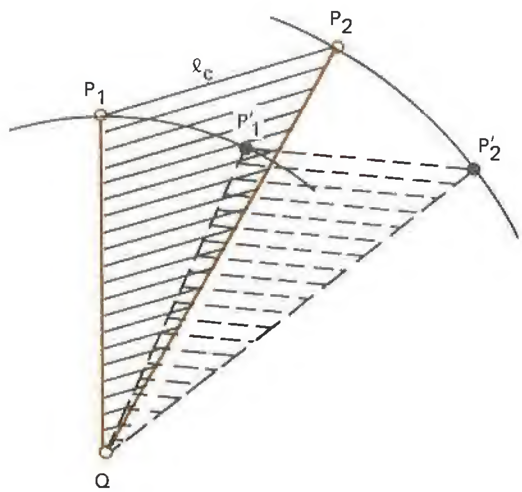


Fig. 4-4 Pure Rotation

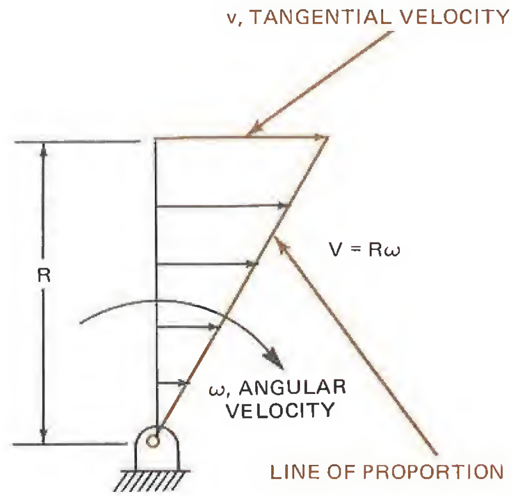


Fig. 4-5 Line of Proportion

The idea that the link is in pure rotation **only** is better visualized if we consider that the triangle formed by QP_1P_2 has *rotated* about the point Q to a new position where the triangle is $QP'_1P'_2$, as shown in figure 4-4.

From the preceding, we see that even though the motion of a body is very complex, the *instantaneous* motion may be considered to be pure rotation. This concept has several uses, one of which is to analyze the motion of various machine members.

In order to develop this idea further, we need to recall a basic fact about bodies which are rotating about a fixed point. The tangentail velocity, V, at the fixed point is zero, and at the points along the body, it varies with distance such that

$$\boxed{V = R\omega}$$

Since ω is a constant at any given instant, and the radius varies linearly along the body, then the tangential velocity is a linear function of the radius. This is the central idea behind the *"line of proportion"* which is used to describe velocities at any point along a link or member, as shown in figure 4-5.

With the centro and line of proportion, the instantaneous angular and linear velocities at any point on a body undergoing motion can be found, provided that the direction of two linear velocities of the body are known, and the magnitude of one is known. The linear or tangential velocities are always perpendicular to a line drawn to the centro, a necessary fact we must remember. As an example of this, consider the body shown in

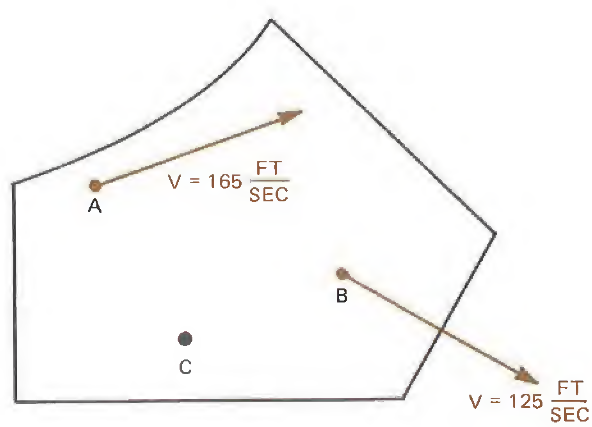
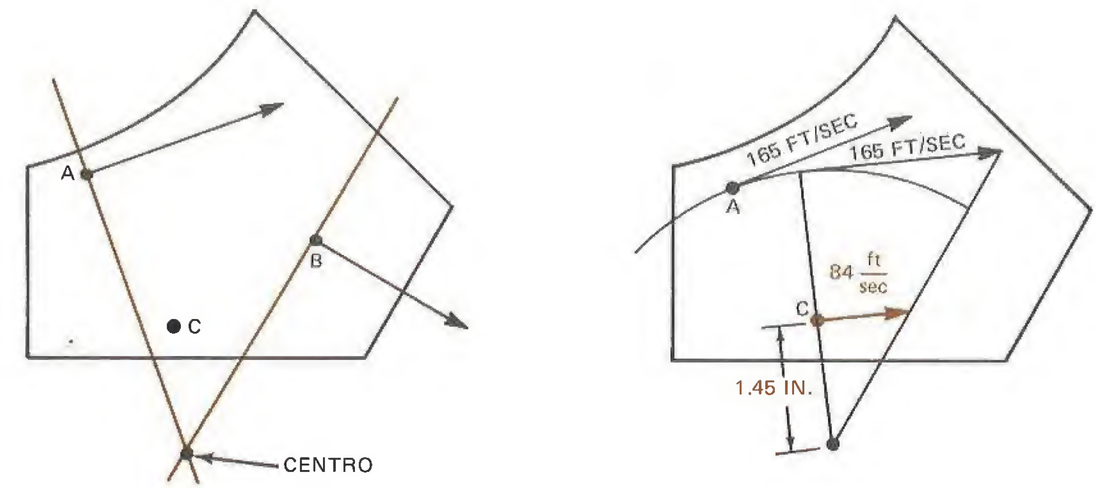


Fig. 4-6 Unknown Velocity



$$V_c = 84\,\frac{\text{FT}}{\text{SEC}}$$

$$V_c = R_c\omega_c$$

$$\omega_c = \frac{V_c}{R_c}$$

$$\omega_c = \frac{84\ \text{FT/SEC}}{(1.45\ \text{IN.})(\text{FT}/12\ \text{IN.})}$$

$$\boxed{\omega_c = 695\,\frac{\text{RAD}}{\text{SEC}}}$$

Fig. 4-7 Velocity Measurement

figure 4-6, where the linear velocities at points A and B are known and the velocity at point C is desired.

The first step in finding the velocity at point C is to locate the centro by drawing two lines through points A and B which are perpendicular to the direction of the velocities. Their point of intersection is the centro. Then the velocity at A *or* B is shifted along its radius arc so that the starting end of the velocity vector is in line with point C and the centro. Now we draw the line of proportionality from the centro to the tip of the velocity vector and *measure* the linear velocity at point C. Using the relation previously given, we can also find the angular velocity at C if it is desired. This procedure is illustrated in figure 4-7.

In finding the centro of a simple mechanism, there are four primary types that will occur. These are: (1) rotating cranks, or links, (2) moving bodies with two known linear velocities, (3) rolling bodies, and (4) sliders. The first two types have already been illustrated, leaving the last two to describe.

A rolling body of the type most frequently encountered in practice will have a point in contact with a second body, with no slippage between the two bodies. At the point of contact, the velocity of the rolling body is zero, since there is no slippage. Therefore, the point of contact forms a centro for the rolling body, as illustrated by figure 4-8, for a cylinder rolling down an inclined plane.

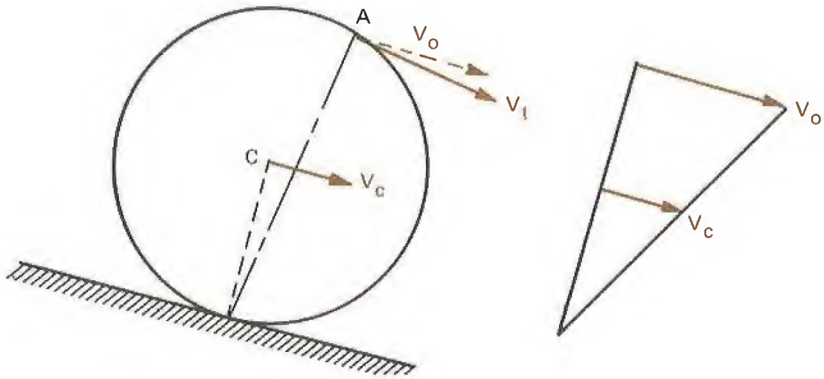


Fig. 4-8 Rolling Cylinder

If the linear velocity and direction at some point A are known, the methods previously described may be used to find the velocity at the center of the cylinder V_C, or at any other point in the cylinder. The tangential velocity at any point along the surface of the cylinder may also be found.

The surface on which a *slider* block is moving has three possible infinitesimal surface patterns: flat, convex, or concave, as in figure 4-9. Note that the centro for a slider moving over a flat surface is at infinity, in either direction. For a convex or concave surface, the centro is always *inside* the arc.

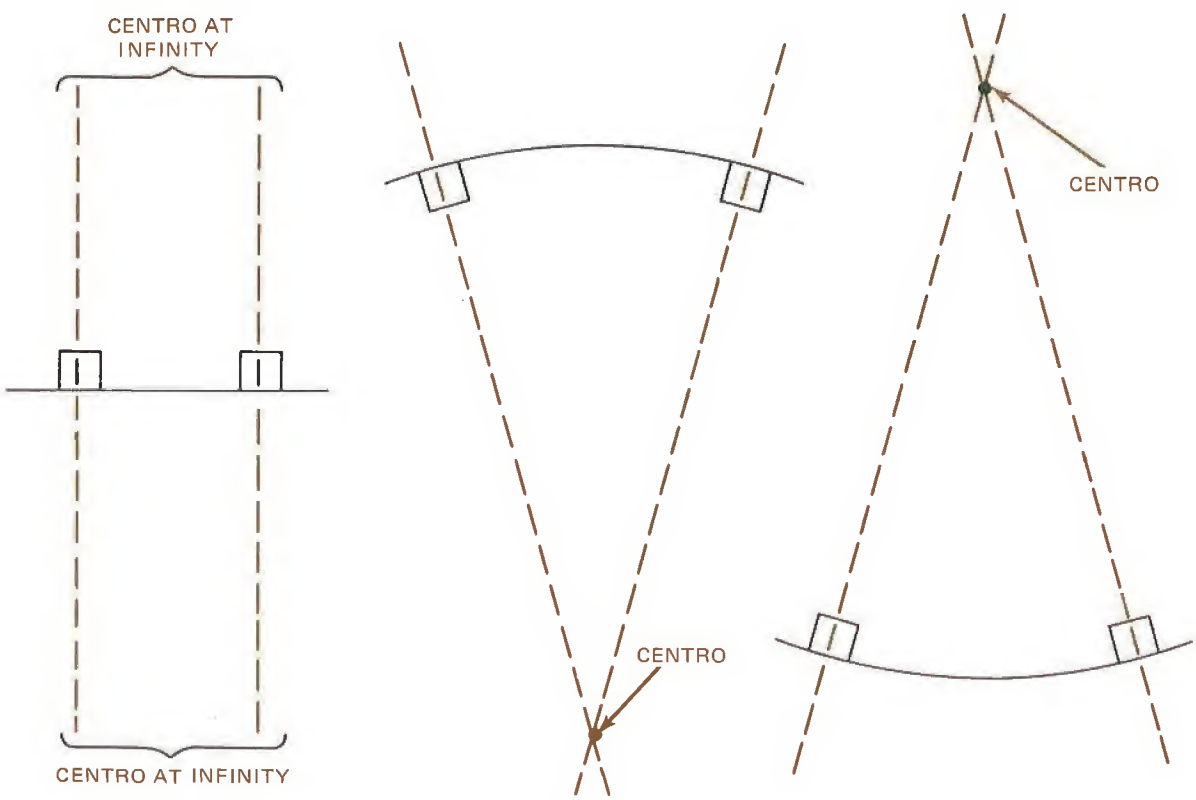


Fig. 4-9 Slider Block Centros

The analysis of centros for mechanisms which are more complex than those previously discussed becomes quite complicated and a good reference text should be consulted when necessary.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Shaft hangers with spacers
2 Bearing holders with bearings
2 Shafts 4 x 1/4
2 Collars
1 Shim
2 Clamps
1 Lever arm 4 in. long with 1/4 in. bore hub
1 Lever arm 1 in. long with 1/4 in. bore hub
2 Straight links 6 in. long
1 2-3 in. gear
2 Sheets of linear graph paper
3 Sheets of drafting paper
1 Pair dividers
1 Engineer's scale
1 Drafting pencil
1 Rectilinear potentiometer
1 Rotary potentiometer
1 DC power supply
1 X-Y or strip-chart recorder

PROCEDURE

1. Check all of your instruments and components to be sure that they are undamaged.
2. Assemble the components as shown in figure 4-10. (See also page 28.)

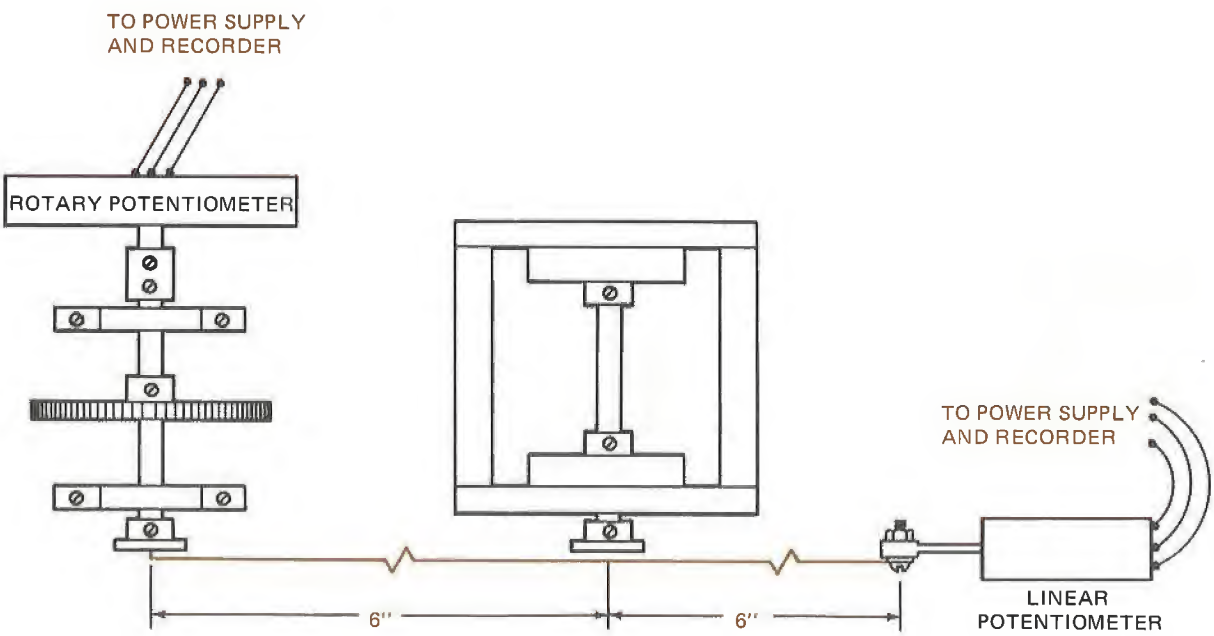
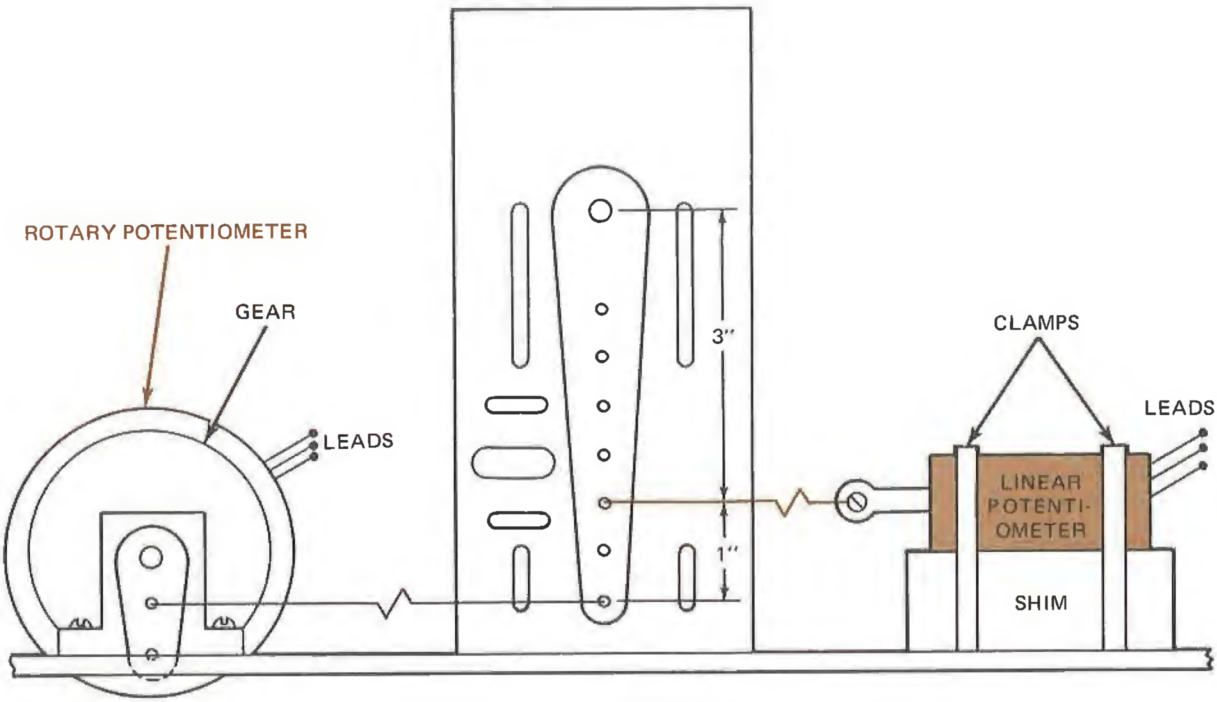


Fig. 4-10 Experimental Equipment



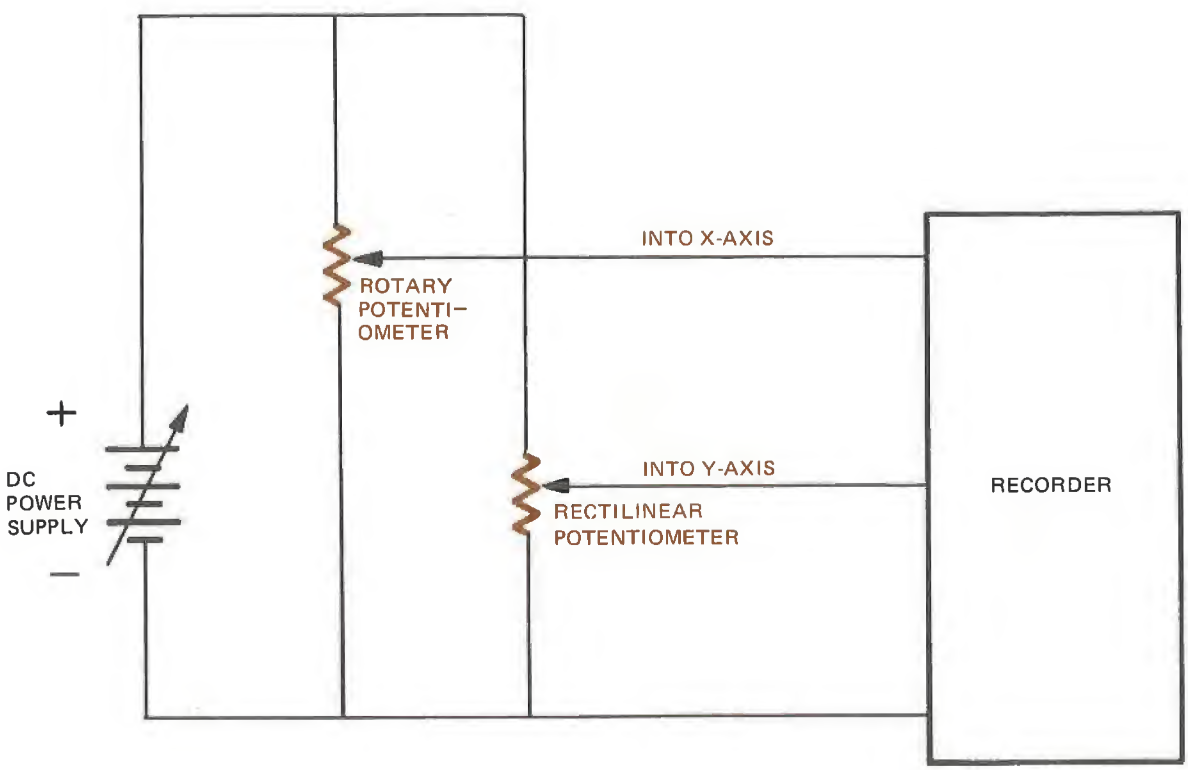


Fig. 4-10 Experimental Equipment (Cont'd)

3. Turn the recorder on first, then turn on the power supply.
4. Adjust the controls on the power supply and recorder to obtain a 6-in. display on the X-axis for one cycle of the mechanism.
5. Adjust the drive link to a 45° angle clockwise from the positive Y-axis and mark the resulting displacement of the rectilinear potentiometer on the recorder plot.
6. Now turn the mechanism through one complete cycle, and obtain the displacement plot on the recorder.
7. Using the mark made in step 5 and the displacement curve from step 7, find the instantaneous tangential and angular velocity for the connecting link at a point 2.5 inches from the follower link, using the methods of figure 4-3 and figure 4-5. It will be to use the method of centros and the line of proportionality.
8. Repeat step 7 for a point 3.5 inches from the follower link.
9. Repeat step for the pin joint connecting the driver and connecting link.
10. Duplicate steps 5 through 7 for a driver angle 60° clockwise from the positive Y-axis.

ANALYSIS GUIDE. What factors do you think affected the accuracy of this experiment's results? Was there any tendency for the linear potentiometer slider to bind at any point in its travel? What effect would this have on the instantaneous angular velocity curve for the follower link?

PROBLEMS

1. Find the instantaneous linear velocity, in ft/sec, and angular velocity, in rad/sec, at point B in figure 4-11 for a linear velocity at point A of 1 mile/hour. The body is rotating about point O.

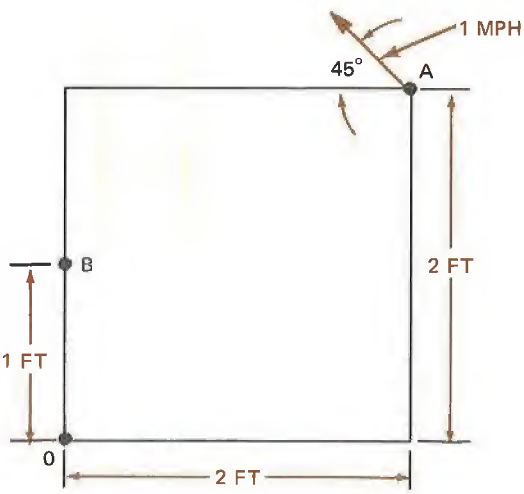


Fig. 4-11 Body for Problem 1

2. For the body in figure 4-12, find the instantaneous linear and angular velocities at point C.

3. If a rotating link has a constant angular velocity of 3π RPM, what is the tangential velocity in ft/sec at distances from the center of rotation of: 0.5 ft, 1 ft, 1.25 ft, 1.53 ft, and 2 ft?

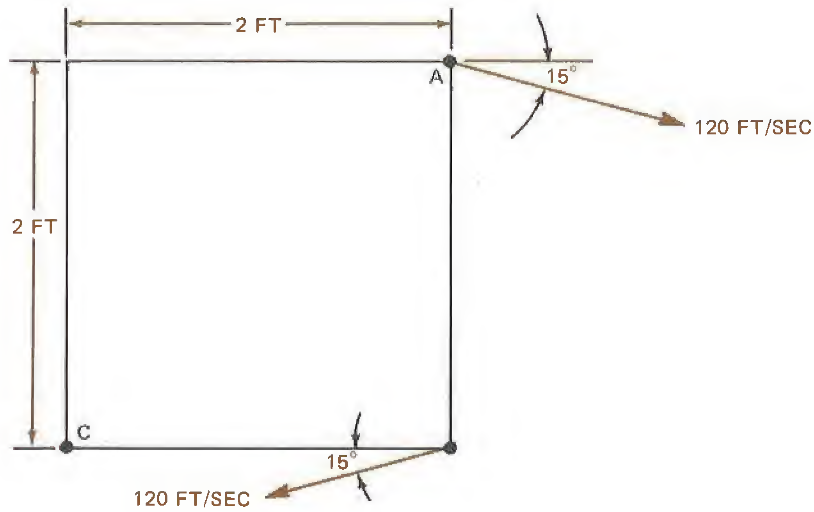


Fig. 4-12

4. For the cylinder rolling down an inclined plane in figure 4-13, with no slippage, what is the instantaneous tangential velocity at point A? What is the instantaneous angular velocity? The cylinder radius is 1 ft.

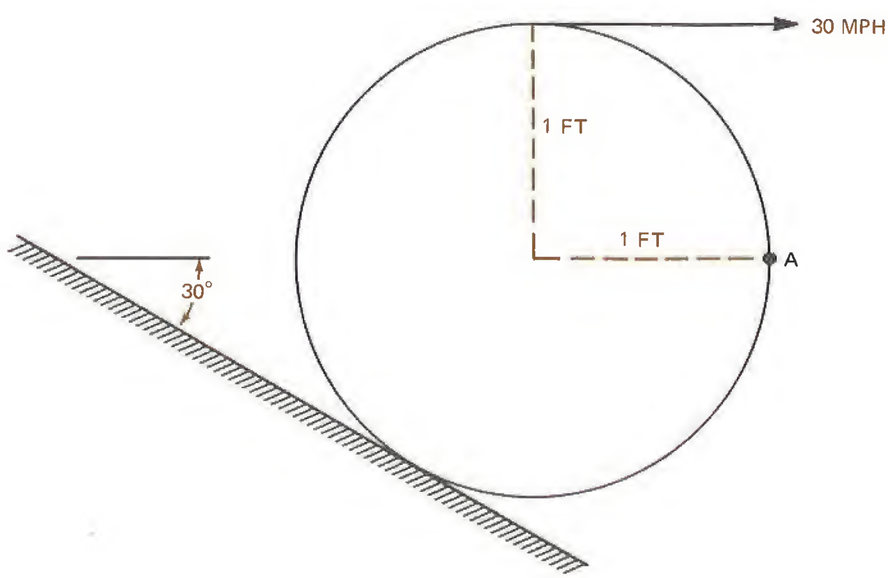


Fig. 4-13 Inclined Plane

5. If the 10-ft-long ladder in figure 4-14 has an instantaneous downward velocity of 50 ft/sec at its upper end, what is the instantaneous tangential and angular velocities at the mid point of the ladder?

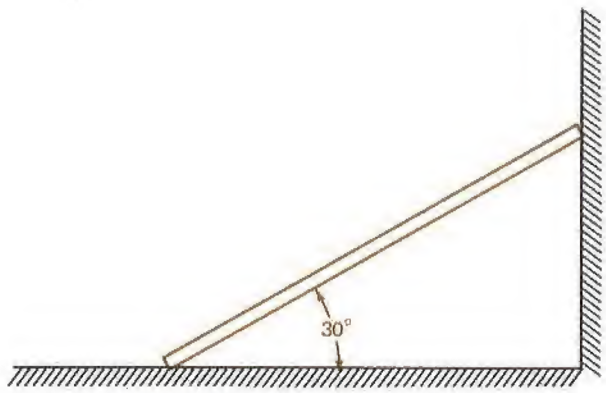


Fig. 4-14 Sliding Ladder

experiment 5 SLIDER-CRANK CHARACTERISTICS

INTRODUCTION. One of the common linkage types is the slider-crank mechanism. In this experiment we shall examine the displacement, velocity, and acceleration characteristics of this mechanism.

DISCUSSION. Slider-crank mechanisms are probably one of the more common mechanisms which people encounter in their daily activities. Four cycle engines usually have from one to eight slider-crank linkages in the form of pistons, connecting rods, and the crankshaft. Types encountered in industry include shapers, which have a quick return feature, and power hacksaws, also having a quick return stroke. The slider may be driven by a four-bar linkage to give a quick return, or the regular slider-crank may be rearranged and a fifth link added for a quick return.

If mathematical equations are derived to give expressions for the displacement S, velocity V, and acceleration, a, of the slider, they result in the following equations of motion for the linkage in figure 5-1.

$$S = \ell_1 (1 - \cos\theta) + \ell_c \left[1 - \sqrt{1 - \left(\frac{\ell_1}{\ell_c}\right)^2 \sin^2\theta} \right]$$

$$V = \frac{dx}{dt} = \ell_1 \omega \left[\sin\theta + \frac{\ell_1}{2\ell_c} \left(\frac{\sin 2\theta}{1 - \left(\frac{\ell_1}{\ell_c}\right)^2 \sin^2\theta} \right) \right]$$

$$a = \ell_1 \omega^2 \left[\cos\theta + \frac{\frac{\ell_1}{\ell_c} \cos^2\theta \left[1 - \left(\frac{\ell_1}{\ell_c}\right)^2 \sin^2\theta \right] - 1/4 \left(\frac{\ell_1}{\ell_c}\right)^3 \sin^2 2\theta}{\left[1 - \left(\frac{\ell_1}{\ell_c}\right)^2 \sin^2\theta \right]^{3/2}} \right]$$

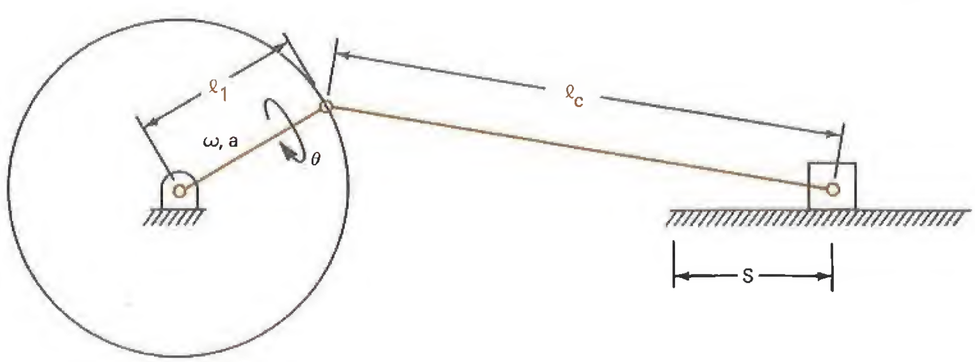


Fig. 5-1 A Slider-Crank

These equations can be used to find the *exact* displacement, velocity, and acceleration for an angle θ. The best way to do this is to use a computer to solve the equations for θ increments which are fairly small, since solving with a slide rule would be very time-consuming.

If an exact solution is not required, then the preceding equations can be simplified, by making approximations and using a power series and binomial expansions, into the following equations:

$$\boxed{S = \ell_1 (1 - \cos\theta) + \frac{\ell_c}{2}\left(\frac{\ell_1}{\ell_c}\right)^2 \sin^2\theta} \quad \textbf{(5.1)}$$

$$\boxed{V = \ell_1 \omega \left(\sin\theta + \frac{\ell_1}{2\ell_c} \sin 2\theta\right)} \quad \textbf{(5.2)}$$

$$\boxed{a = \ell_1 \omega^2 \left(\cos\theta + \frac{\ell_1}{\ell_c} \cos 2\theta\right)} \quad \textbf{(5.3)}$$

The thing most apparent about equations 5.1, 5.2, and 5.3 is that they are much easier to solve than the equations they were obtained from. Solution by hand is now more easily done and does not require so much time.

Also, the three equations are a function of the crank lengths and connecting link length. As the crank length increases or the connecting link length decreases, S, V, and a become larger.

However, the three equations are still too long to give a very quick knowledge of what the characteristic curves should look like. Graphic analysis will give quick results and have relatively small error if reasonable care is taken in deriving the motion curves. For typical problems where great accuracy is not required, graphic analysis often works very well.

In figure 5-1 the slider-crank is a simple type. The crank rotates about a center line which is on a horizontal line passing through the pin joint on the slider. If the crank base is elevated as in figure 5-2, the equations of motion become more complex. The displacement, S, is now a function of ℓ_1, ℓ_c, θ, and y, where y is the displacement of the slider base above the horizontal line through the slider pin joint.

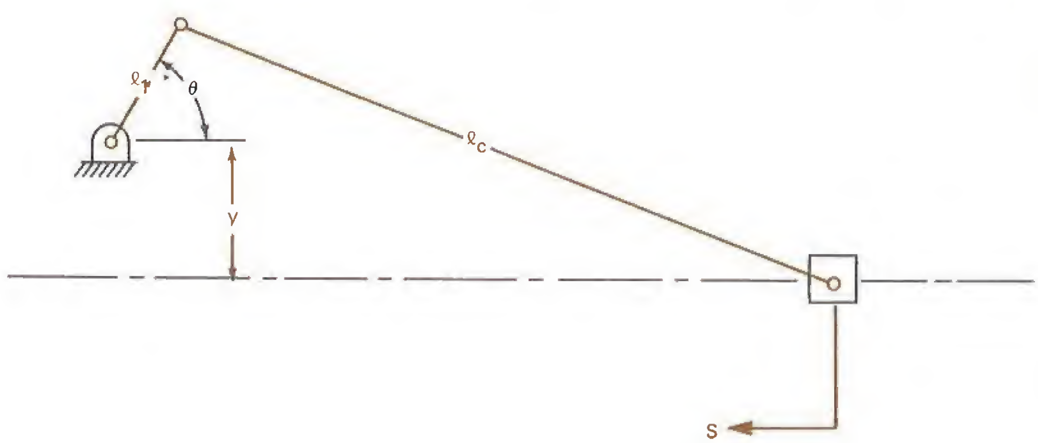


Fig. 5-2 Elevated Crank Base

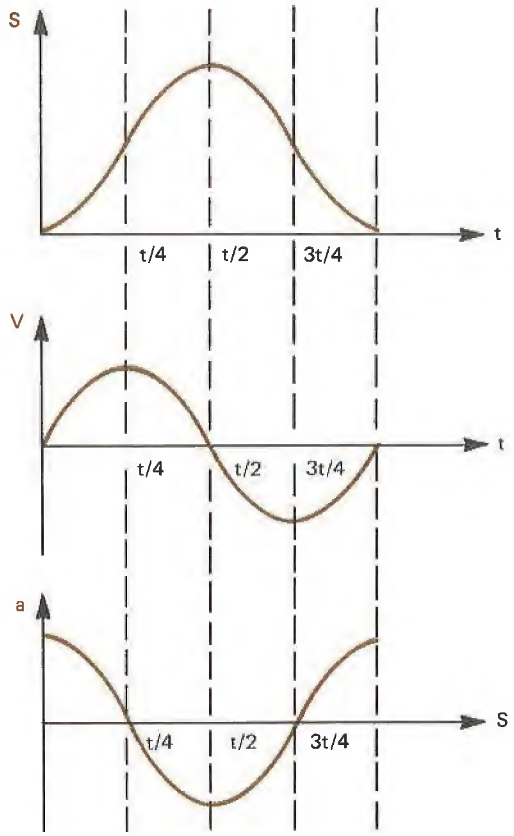


Fig. 5-3 Slider Crank Motion Curves

Graphic analysis for the linkage in figure 5-2 is much faster and easier than attempting to solve the equations of motion for S, V, and a. Figures 5-1 and 5-2 will both give us curves similar to those in figure 5-3. However, in figure 5-2, the linkage will give a quick-return type of motion curve as y becomes larger.

From figure 5-3 we can observe several features which are characteristic of differentiation. As the slider first begins its movement at $t = 0^+$, there is no velocity; but acceleration is at its maximum value. Then, since velocity is the slope of the displacement curve at a point, the velocity increases, since the slope of the displacement curve is increasing steadily to its maximum value at the point t/4, where the slope of the displacement curve is greatest. At the point of peak displacement, t/2, the slope is zero, so the velocity is also zero at this point.

Before zero velocity, from t/4 to t/2, the slope of the displacement curve is decreasing, so velocity also decreases, and continues to decrease to its minimum value at the point 3t/4, where the slope velocity begins increasing again. Velocity now increases to a value of zero displacement, as the slope here is also zero.

This process of reasoning can be applied with some practice to give sketches of velocity and acceleration extremely rapidly and to check on the results of more formal graphic analysis.

A slider-crank rearranged into a quick-return mechanism as in figure 5-4 will not give motion curves which are symmetrical.

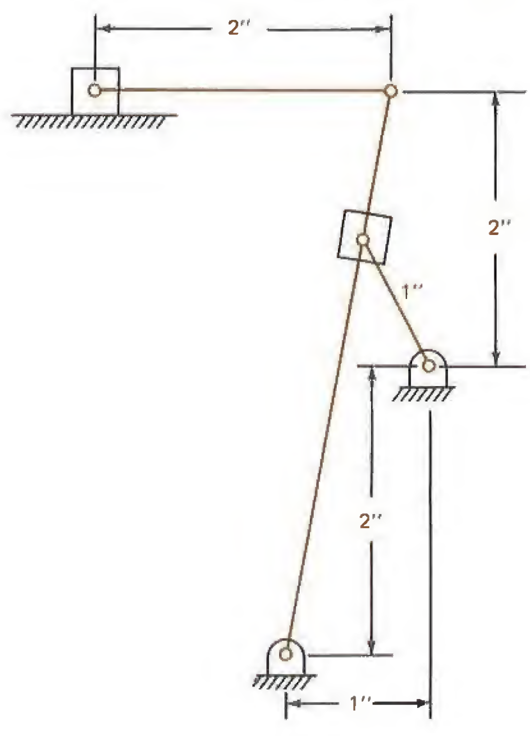
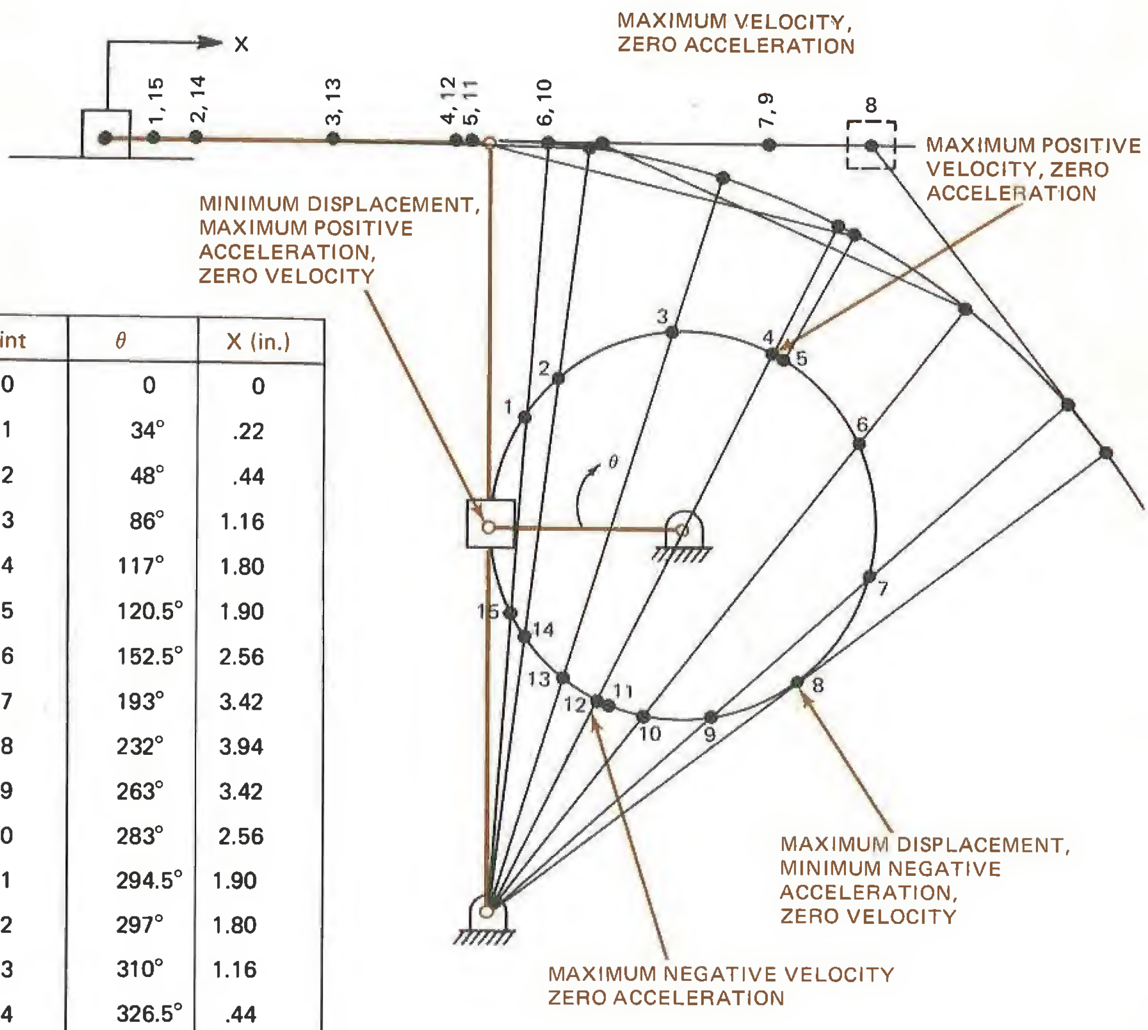


Fig. 5-4 Quick-Return Linkage

As an example of graphic analysis, we shall find the motion curves for the linkage of figure 5-4.

The first step is to draw the linkage with the arcs of motion of the fixed end links included. Then the linkage is drawn in various configurations with emphasis on the limiting positions and the two points where there is a 90° angle between the driver crank and the link it is connected to. The significance of the last statement can be better understood if we realize that the limiting positions are where maximum acceleration occurs, and the "90° angles" are where the maximum velocity occurs, for a regular crank-slider.

For the quick-return linkage of figure 5-4, however, the limiting positions an "90° angle" occurs at the same point and the slider has its maximum velocity when the drive link is parallel to the connecting link. The procedure for this case is shown in figure 5-5.



Point	θ	X (in.)
0	0	0
1	34°	.22
2	48°	.44
3	86°	1.16
4	117°	1.80
5	120.5°	1.90
6	152.5°	2.56
7	193°	3.42
8	232°	3.94
9	263°	3.42
10	283°	2.56
11	294.5°	1.90
12	297°	1.80
13	310°	1.16
14	326.5°	.44
15	335°	.22

Fig. 5-5 Finding Displacement

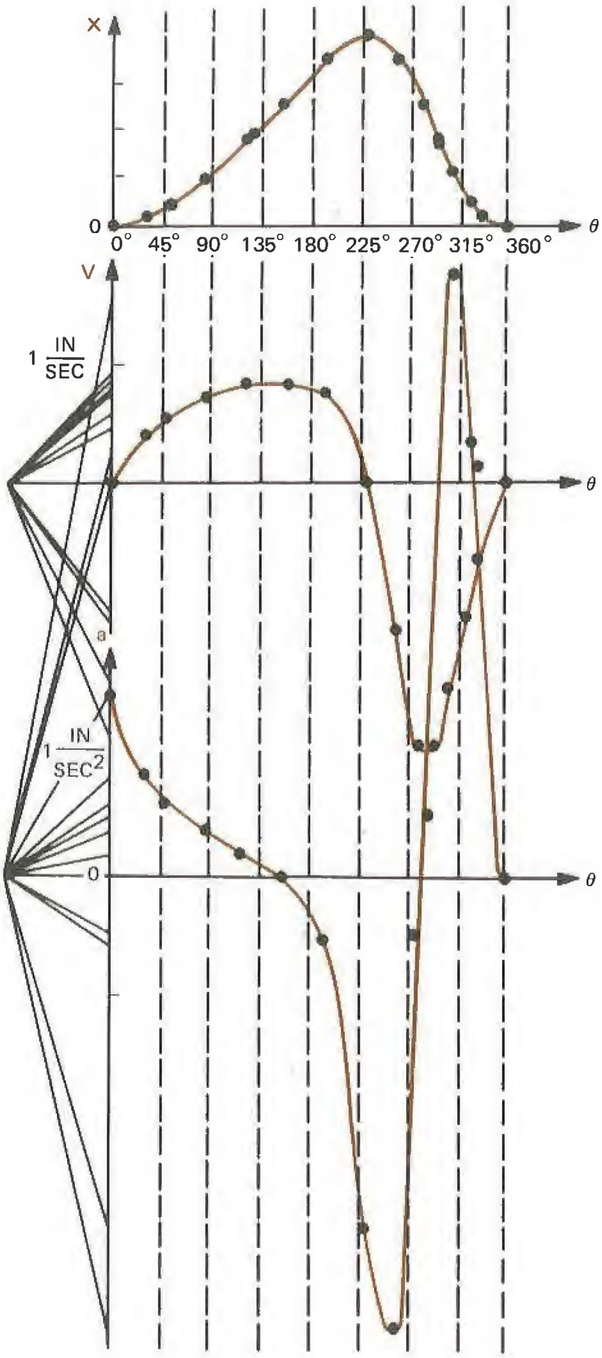


Fig. 5-6 Motion Curves

The procedure now is the same for all linkages after θ and S increments have been found. The values of θ and S are plotted or, if ω is known, values of t and S are plotted. This is accomplished by remembering that:

$$\boxed{\theta = \omega t}$$

Therefore,

$$\boxed{t = \frac{\theta}{\omega}}$$

The displacement plot is graphically differentiated to obtain the velocity plot, which is itself differentiated to find the acceleration curve. These steps are illustrated in figure 5-6 for the displacement values given in figure 5-5.

As seen from figure 5-6, the motion curves are not symmetrical. This is a reasonable result for the linkage from which the curves were obtained, since the slider does not undergo symmetrical motion during its cycle. While the crank driver is turning from 0° to 180°, the slider is moving fairly slowly and the velocity and acceleration are not too great. However, in the part of the curve from about 180° to 360°, the acceleration has a *large* change in value which will be undesirable if the driver has a large ω.

MATERIALS

1 Breadboard with legs and clamp
2 Bearing plates with spacers
2 Bearing mounts with bearings
1 Rotary potentiometer
1 Linear potentiometer
1 X-Y or strip-chart recorder

2 Shafts 4 X 1/4 in.
4 Collars
4 2-1/2 in. shaft hangers, 2 with bearings
1 Lever arm 1 in. long with 1/4 in. bore hub
2 Wire links 6 in. long
1 Rigid shaft coupling
1 DC power supply
1 Compass
1 Divider
1 Engineer's scale
1 Straightedge
1 Drafting pencil
1 Link, 1 in. long

PROCEDURE

1. Inspect each of the instruments and components to be sure they are undamaged.

2. Connect the components as shown in figure 5-7.

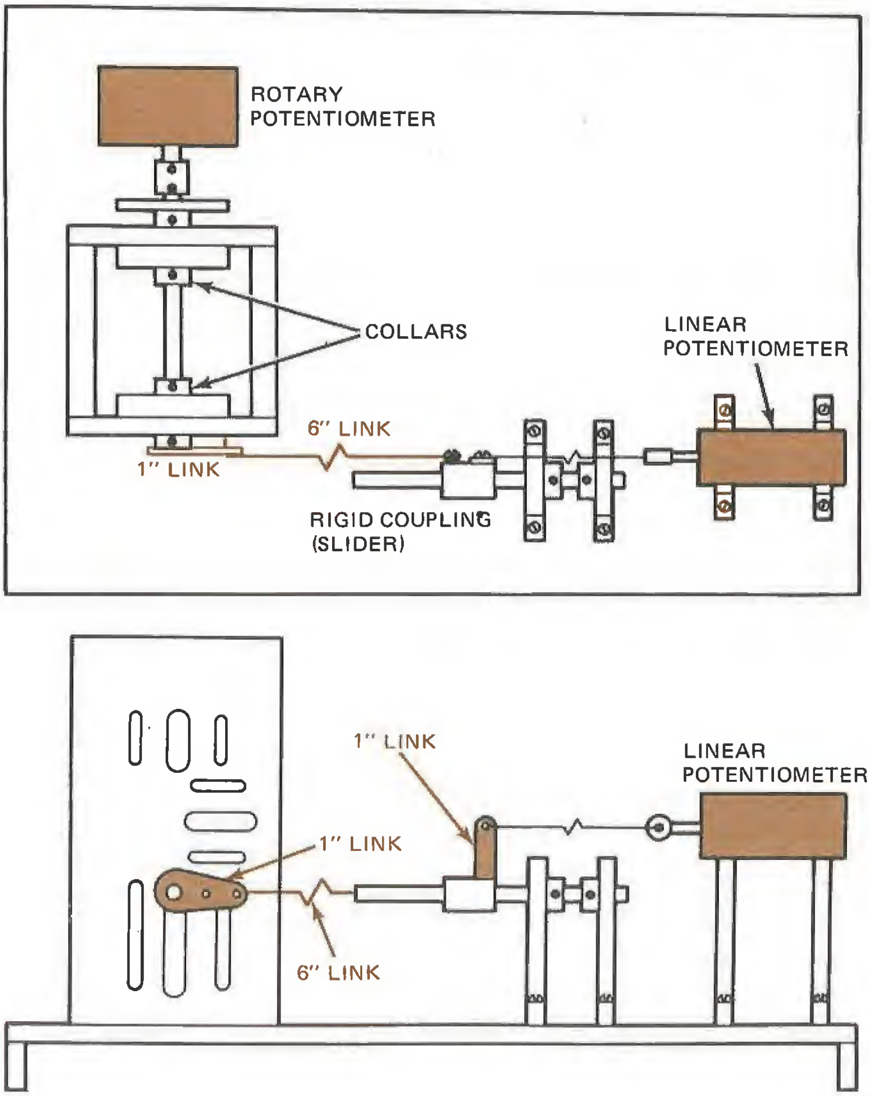


Fig. 5-7 Experimental Equipment

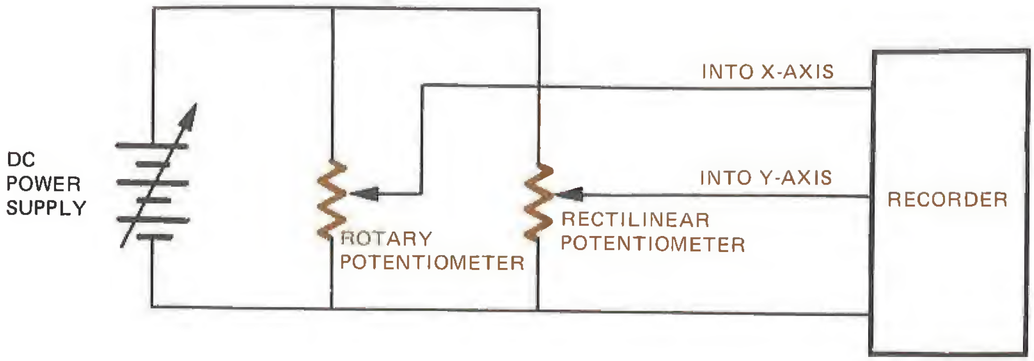


Fig. 5-7 Experimental Equipment (Cont'd)

3. Turn the recorder on first, then turn the DC power supply on.
4. Adjust the controls to obtain an approximate 6-in. display on the X-axis and a 4-in. display on the Y-axis, for one revolution of the crank.
5. Turn the crank through 360° and obtian a plot of displacement versus input angle.
6. Graphically differentiate the displacement plot to obtain the velocity plot for the linkage.
7. Graphically differentiate the velocity plot to obtain the acceleration curve.
8. *Integrate,* by graphic techniques, the acceleration curve to get first the velocity plot and then the displacement curve.
9. Change the crank length to 1/2 inch and repeat step 5.
10. Change the connecting link length to 5 inches and repeat step 5 with a crank length of 1/2 in.

ANALYSIS GUIDE. Explain in your own words how and why the motion curves obtained in steps 9 and 10 varied from step 5. Consideration should be given to equations 5.1, 5.2, and 5.3. Did step 9 or 10 vary the most from step 5? Explain.

PROBLEMS

1. Draw the linkage shown in figure 5-8 and obtain the motion curves by graphical analysis.

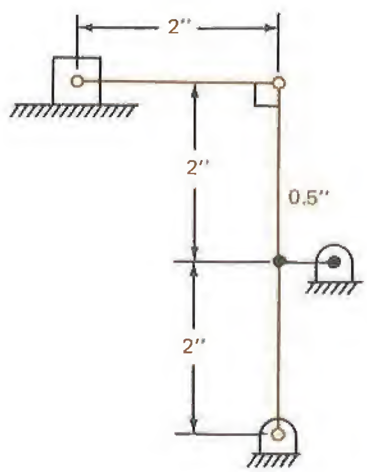


Fig. 5-8 Linkage for Problem 1

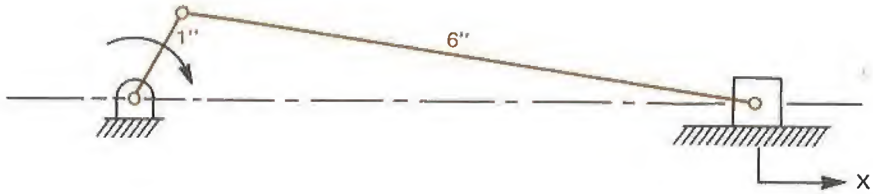


Fig. 5-9 Slider-Crank

2. Draw the linkage shown in figure 5-9 and obtain the *motion curves.*
3. Compare the curves in problem 2 with those from the actual mechanism. Why do the acceleration curves show more variance than the displacement or velocity curves?
4. Would the linkage in figure 5-10 make an extremely efficient rock crusher? If not, why not?

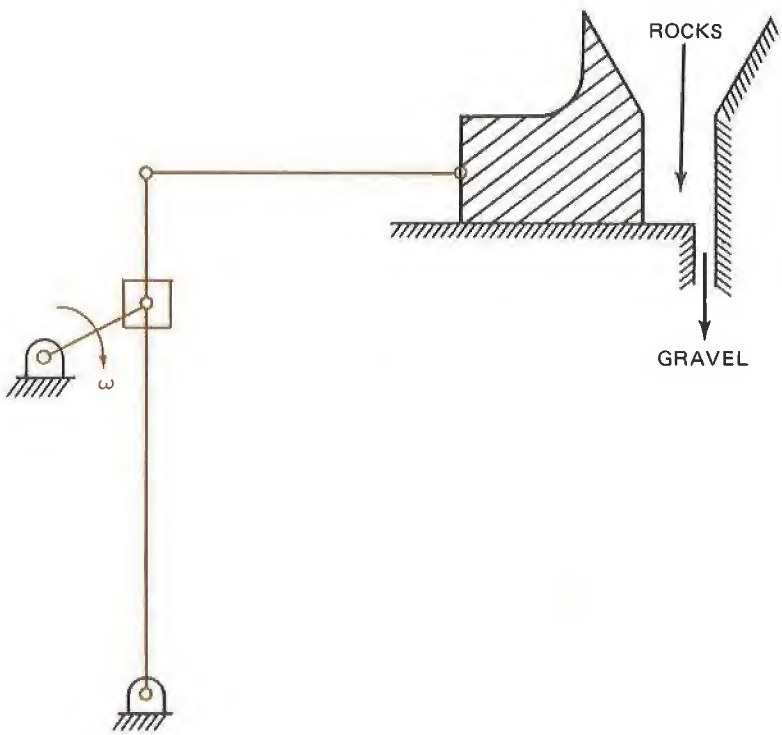


Fig. 5-10 Rock Crusher

5. Draw simple sketches of the linkage in figure 5-10, modified to be a more efficient rock crusher. Show at least two different ways the mechanism could be modified.

experiment 6 VELOCITY AND ACCELERATION POLYGONS

INTRODUCTION. Complex problems can be rapidly solved by the use of velocity and acceleration polygons. During this exercise we will investigate this method.

DISCUSSION. In order to work with velocity and acceleration polygons, we first need to develop some basic conditions for using vectors. You will recall that vectors are distinguished from scalars by the fact that vectors have **both direction and magnitude,** while scalars have **only magnitude.** A good example of this is one which is commonly misused, that of velocity and speed. Velocity is actually a vector possessing both direction and magnitude. People use the term velocity very loosely and say, "The airplane had a velocity of 600 miles per hour." This is technically incorrect, since there is no direction of motion included in the statement. Technically correct usage would have to be either, "The airplane had a velocity of 600 miles per hour due south," or, "The airplane had a speed of 600 miles per hour." The last sentence would be correct since velocity has both magnitude and direction described, and speed needs no direction as it is a scalar quantity.

Relative and absolute velocity also need to be understood in order to use velocity polygons effectively. **Absolute** velocity is a velocity found with respect to a **fixed** point. **Relative** velocity is that velocity which is observed from a **moving** point. This can be illustrated by a simple example.

If a train is moving due north past a train station at a speed of 60 miles per hour, its velocity vector relative to a stationary observer on the station platform would be 60 mph due north. In this case, the velocity observed is *both* relative and absolute. An observer who is not stationary, however, will see a different relative velocity than the stationary observer.

For an observer riding in another train on tracks parallel to the first train, moving with a speed of 50 mph, there are two possible values for the relative velocity. If we define *sense* as the *direction* in which an object is moving, then positive or negative signs are assigned to velocity vectors according to the direction in which they are moving with respect to the reference direction.

In the first train's direction of motion (due north) is assigned a *positive* direction, then movement or components of movement in a northern direction are positive. Movement or components of movement in a *southern* direction are then assigned *negative* signs. This is illustrated by the vector diagram of figure 6-1 for several vectors.

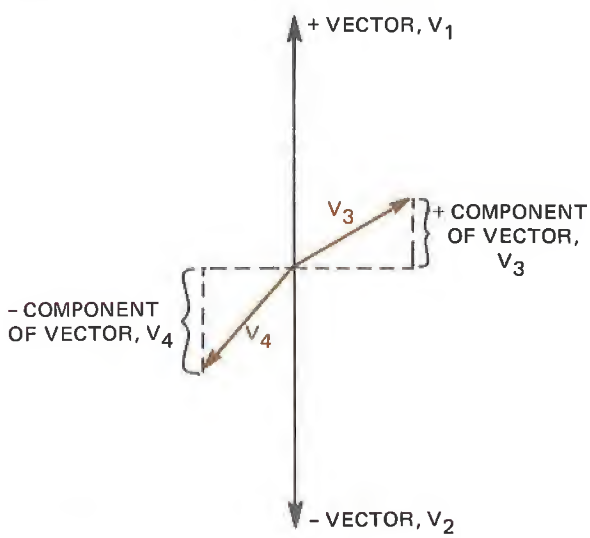


Fig. 6-1 Vector Signs

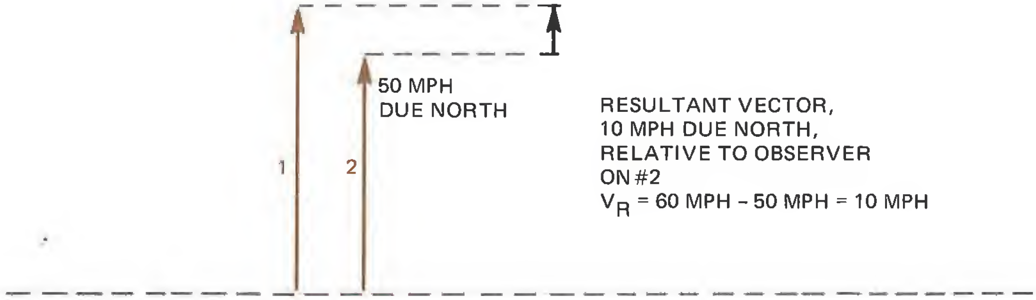


Fig. 6-2 Vector Difference

Fig. 6-3 Vector Difference

The relative velocity, termed the *resultant* vector, seen by the observer on the second train, can thus have two different values depending on the direction of travel of the trains.

If the first is heading due north at 60 mph and the second is heading due north at 50 mph, the *resultant* velocity, V_r, is found by *vector difference* as shown in figure 6-2.

If the first train is moving due north at 60 mph and the second is moving due south at 50 mph, the relative velocity seen by an observer on the second may again be found by vector difference. This case is illustrated in figure 6-3.

This procedure for finding resultant vectors becomes slightly more complex for vectors which are not parallel to each other.

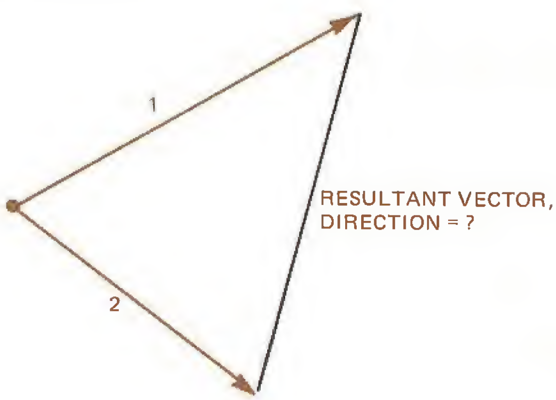


Fig. 6-4 Nonparallel Vectors

There are several methods of finding the resultant velocity in this situation. One is to draw each vector from a common base and then draw the resultant vector between the two. This is illustrated in figure 6-4.

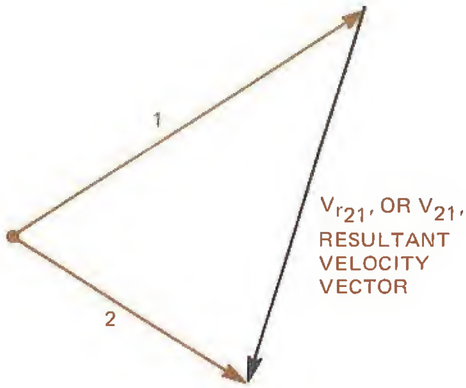


Fig. 6-5 Resultant Velocity

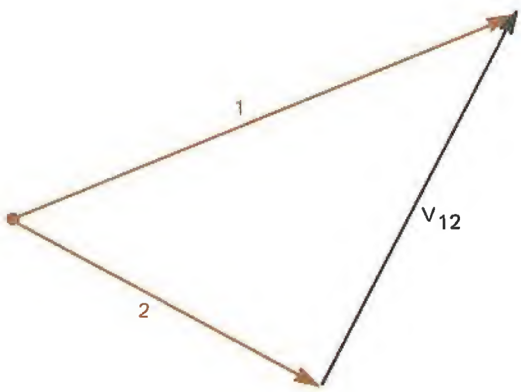


Fig. 6-6 Resultant Velocity

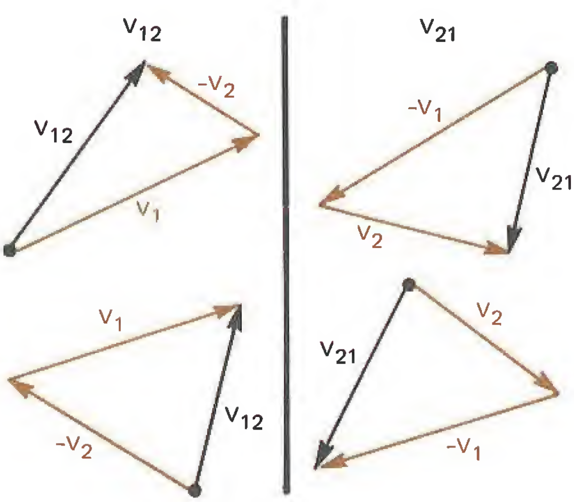


Fig. 6-7 Resultant Velocity

The arrowhead has been left off the resultant vector in figure 6-4 because we did not state what it was to be *relative to.* The most common nomenclature system is that if the resultant vector, V_r, is to be the velocity of *2 relative to 1,* then V_r is given the subscript 21 to indicate the sense, with the direction shown in figure 6-5.

If the velocity of *1 relative to 2* is desired, the subscripts are then 12, with the sense given by figure 6-6 from 2 to 1.

The second method for finding V_r, commonly used in drafting techniques, is to *add* the vectors by placing the *origin* of one vector at the *head* of the other vector. The resultant vector is then drawn from the origin to the tip of the second vector. This is shown in figure 6-7 for V_{12} and V_{21}. The sense of one of the vectors is changed according to whether V_{12} or V_{21} is desired. If V_{12} is desired, then the sign on V_2 is changed. If V_{21} is desired, then the sign on V_1 is changed. The order in which the vectors are placed makes no difference, as shown in figure 6-7.

Vector equations may be written for the relative velocities in figure 6-7 which are extremely simple, but convey the information in much shorter form, if the direction and magnitude of the two vectors are known. These equations are given as equations 6.1 and 6.2

$$\boxed{V_1 = V_2 + V_{12}} \qquad (6.1)$$

$$\boxed{V_2 = V_1 + V_{21}} \qquad (6.2)$$

The preceding method for finding the resultant vector may also be extended to three or more vectors, forming what is termed a *vector polygon.* This is illustrated by the example of the four vectors in figure 6-8.

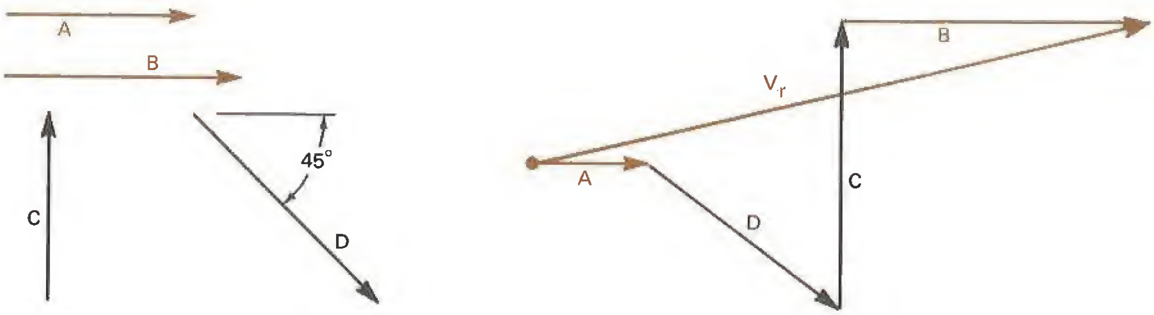


Fig. 6-8 Vector Polygon

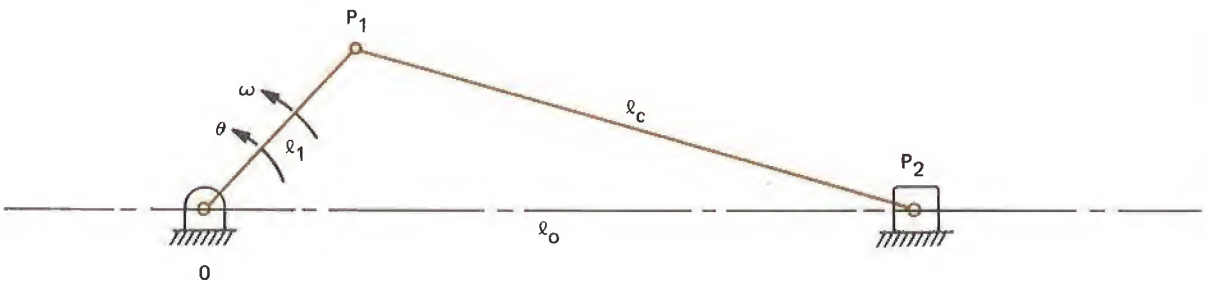


Fig. 6-9 A Slider-Crank

Using the method of resultant vectors, let's consider the slider-crank linkage in figure 6-9.

If we try to find the velocity of point P_2 at some angle θ, one way to do it would be to obtain the displacement curve and graphically differentiate the curve to find the velocity. An even *faster* way is to use the method of *velocity polygons,* making it unnecessary to find the displacement curve.

Since ω and ℓ_1 of the crank are known, then the tangential velocity at point P_1 can easily be determined. Also, the direction of velocity of point P_2 is known since it travels in a straight, fixed path. The direction of V_{21} is known since it is perpendicular to the vector V_2. With this information we can draw the vector polygon of figure 6-10, measure the magnitude of V_{21}, and scale it to the correct value.

If we had known the magnitude of V_2 instead of that for V_1, we could have reversed the process to find the magnitude of V_1.

When acceleration is desired for point P_2, at some θ, a vector polygon is again used, although it becomes more complex, since the tangential acceleration (a_t) and the normal acceleration (a_n) components must be included to find a_2. The first step in finding a_2 is to compute all of the known accelerations and decide their directions.

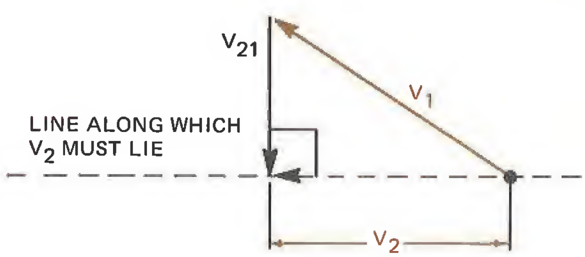


Fig. 6-10 Slider-Crank Velocity Polygon

Because the angular velocity of the driver crank is *uniform,* its tangential acceleration (a_{t1}) is

$$a_{t1} = \frac{dV_1}{dt} = \frac{d(\ell_1 \omega)}{dt} = \frac{d\ (\text{constant})}{dt} = 0$$

Therefore, the only acceleration the drive link has is the centripetal acceleration (or normal acceleration), a_{n1}, directed from P_1 to 0 with a magnitude of

$$\boxed{a_{n1} = \frac{V^2}{\ell_1} = \frac{\ell_1{}^2 \omega^2}{\ell_1} = \ell_1 \omega^2} \qquad (6.3)$$

The tangetial acceleration of P_2 with respect to P_1, (a_{t21}) has a direction perpendicular to ℓ_c, but the magnitude is unknown. The magnitude of a_{n21} is

$$a_{n21} = \frac{V_{21}{}^2}{\ell_c}$$

where V_{21} is found from the velocity polygon in figure 6-10.

A vector equation for the acceleration of point P_2 can be written as

$$\boxed{a_2 = a_1 + a_{21}} \qquad (6.4)$$

This equation may be broken up into normal and tangential components:

$$\boxed{\begin{aligned} a_2 &= a_1 + a_{21} = (a_{n1} + a_{t1}) + (a_{n21} + a_{t21}) \\ &= a_{n1} + a_{t1} + a_{n21} + a_{t21} \end{aligned}} \qquad (6.5)$$

But, since a_{t1} in equation 6.5 is zero, it becomes

$$\boxed{a_2 = a_{n1} + a_{n21} + a_{t21}} \qquad (6.6)$$

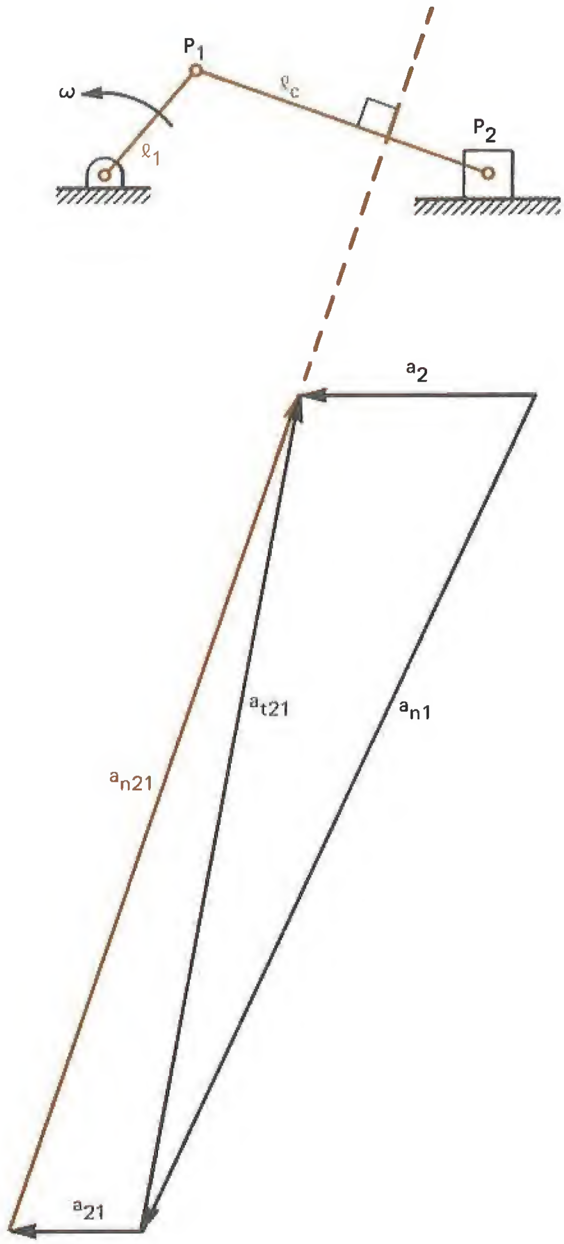


Fig. 6-11 Acceleration Polygon

From the previous work we notice that the direction of each term is known, but the magnitude of a_2 and a_{t21} are unknown.

The acceleration polygon for determining the unknown magnitudes is shown in figure 6-11 for arbitrary values of the known accelerations.

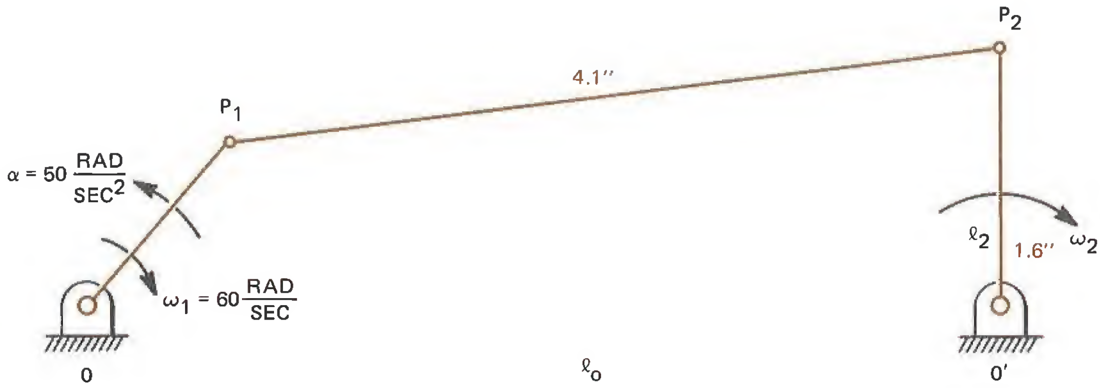


Fig. 6-12 A Four-Bar Linkage

The magnitudes of a_2 and a_{t21} may now be measured from the acceleration polygon of figure 6-11 and then scaled to their actual values.

As another example of the use of velocity and acceleration polygons, let's consider the four-bar linkage shown in figure 6-12. We shall find the acceleration of points P_1 and P_2 and the angular accelerations and velocities of links ℓ_c and ℓ_2.

The first step is to find the velocity polygon. The angular velocity for P_1, ω_{P_1}, is tangential to the path of P_1, or perpendicular to link ℓ_1, and has a magnitude of 60 rad/sec. The direction of ω_2 is perpendicular to link ℓ_2, and the direction of ω_{21} is perpendicular to link ℓ_c. With this knowledge, the only unknown quantities are the magnitudes of ω_2 and ω_{21}, which are found from the velocity polygon of figure 6-13.

The magnitudes of the two unknown velocities are measured and found to be

$$\omega_2 = 55\ \frac{\text{rad}}{\text{sec}},\ \omega_{21} = 19\ \frac{\text{rad}}{\text{sec}}.$$

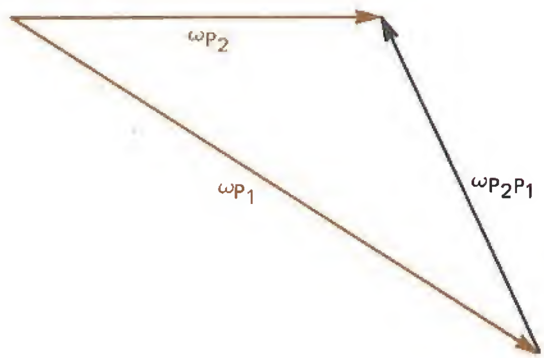


Fig. 6-13 Velocity Polygon

To find the accelerations, the acceleration equation for point P_2 is written as in equation 6.7.

$$\boxed{\begin{gathered} a_2 = a_1 + a_{21} \\ a_{n2} + a_{t2} = a_{n1} + a_{t1} + a_{n21} + a_{t21} \end{gathered}} \qquad \textbf{(6.7)}$$

The magnitudes and directions are now calculated, if possible.

$$a_{n2} = \frac{V_2^2}{\ell_2} = \frac{\ell_2^2\,\omega_2^2}{\ell_2} = \ell_2\omega_2^2$$

$$= (1.6\text{ in.})\left(55\ \frac{\text{rad}}{\text{sec}}\right)^2 = 4840\ \frac{\text{in.}}{\text{sec}^2}$$

(Parallel to link ℓ_2)

$$a_{t2} = \ell_2 \alpha_2 = ?$$

(Perpendicular to link ℓ_2)

$$a_{n1} = \frac{V_1^2}{\ell_1} = \frac{\ell_1^2 \omega_1^2}{\ell_1} = \ell_1 \omega_1^2$$

$$= (1.1 \text{ in.}) \left(60 \frac{\text{rad}}{\text{sec}}\right)^2$$

$$= 3960 \frac{\text{in.}}{\text{sec}^2}$$

(Parallel to link ℓ_1)

$$a_{t1} = \ell_1 \alpha_1 = (1.1 \text{ in.}) \left(50 \frac{\text{rad}}{\text{sec}^2}\right) = 55 \frac{\text{in.}}{\text{sec}^2}$$

(Perpendicular to link ℓ_1)

$$a_{n21} = \frac{V_{21}^2}{\ell_c} = \frac{\ell_c^2 \omega_{21}^2}{\ell_c} = \ell_c \omega_{21}^2$$

$$= (4.1 \text{ in.}) \left(19 \frac{\text{rad}}{\text{sec}}\right)^2 = 1480 \frac{\text{in.}}{\text{sec}^2}$$

(Parallel to link ℓ_c)

$$a_{t21} = \ell_c \alpha_2 = ?$$

Perpendicular to link ℓ_c)

We see that, of the desired quantities, only the magnitudes of a_{t2} and a_{t21} are unknown.

Now the vector addition of the components of a_1 and a_{21} is performed to find a_1 and a_2 as in figure 6-14.

By measurement we find the following values for the accelerations:

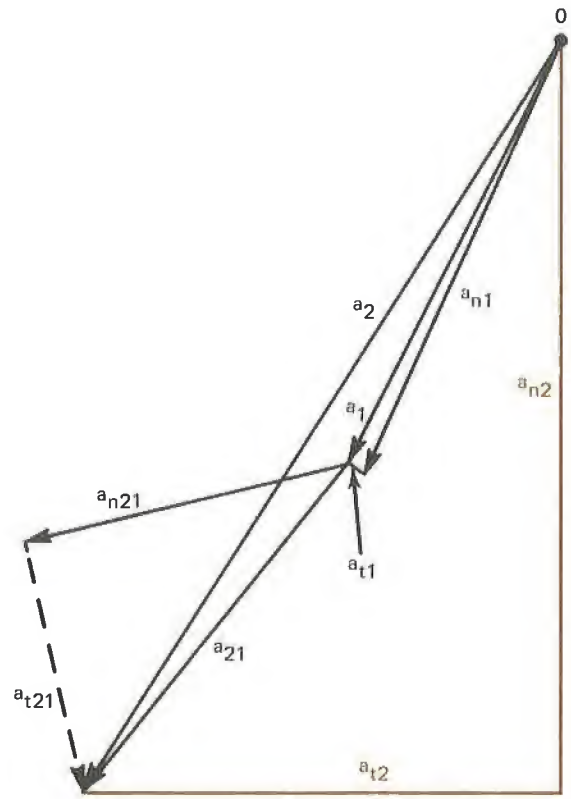


Fig. 6-14 The Acceleration Polygon

$$a_1 = 3965 \frac{\text{in.}}{\text{sec}^2} \qquad a_{21} = 1750 \frac{\text{in.}}{\text{sec}^2}$$

$$a_2 = 5480 \frac{\text{in.}}{\text{sec}^2} \qquad a_{t21} = 920 \frac{\text{in.}}{\text{sec}^2}$$

$$a_{t2} = 2530 \frac{\text{in.}}{\text{sec}^2}$$

Now we can calculate the values of the angular accelerations.

$$\alpha_2 = \frac{a_{t21}}{\ell_c} = \frac{920 \frac{\text{in.}}{\text{sec}^2}}{4.1 \text{ in.}} = 224 \frac{\text{rad}}{\text{sec}^2}$$

$$\alpha_3 = \frac{a_{t2}}{\ell_2} = \frac{2530 \frac{\text{in.}}{\text{sec}^2}}{1.6 \text{ in.}} = 1580 \frac{\text{rad}}{\text{sec}^2}$$

MATERIALS

- 1 Breadboard with legs and clamp
- 2 Bearing plates with spacers
- 2 Bearing mounts with bearings
- 2 Shafts 4 X 1/4 in.
- 4 Collars
- 2 Shaft hangers with bearings
- 1 Lever arm 1 in. long with 1/4 in. bore hub
- 2 Wire links 6 in. long
- 1 Rigid shaft coupling
- 1 DC power supply
- 1 Rotary potentiometer
- 1 Linear potentiometer
- 1 X-Y or strip-chart recorder
- 1 Compass
- 1 Engineer's scale
- 1 Straightedge
- 1 Drafting pencil
- 1 Protractor

PROCEDURE

1. Check all your equipment to be sure it is undamaged.
2. Connect the linkages and instruments as shown in figure 6-15, pages 47, 48.
3. Operate the mechanism for one complete cycle to obtain a displacement curve for the slider.
4. Graphically differentiate the displacement curve to find the numerical values of velocity and acceleration when the drive link, OP_1 is 45° CW from the positive X-axis.

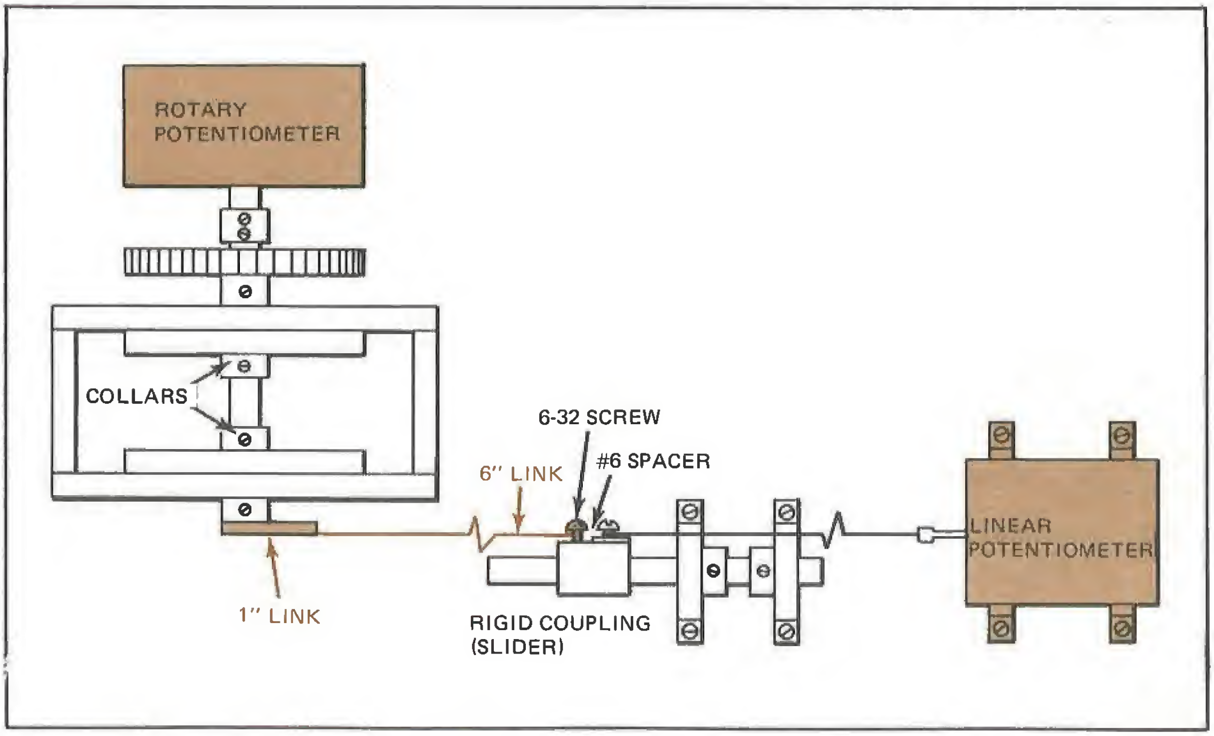


Fig. 6-15 Experimental Equipment

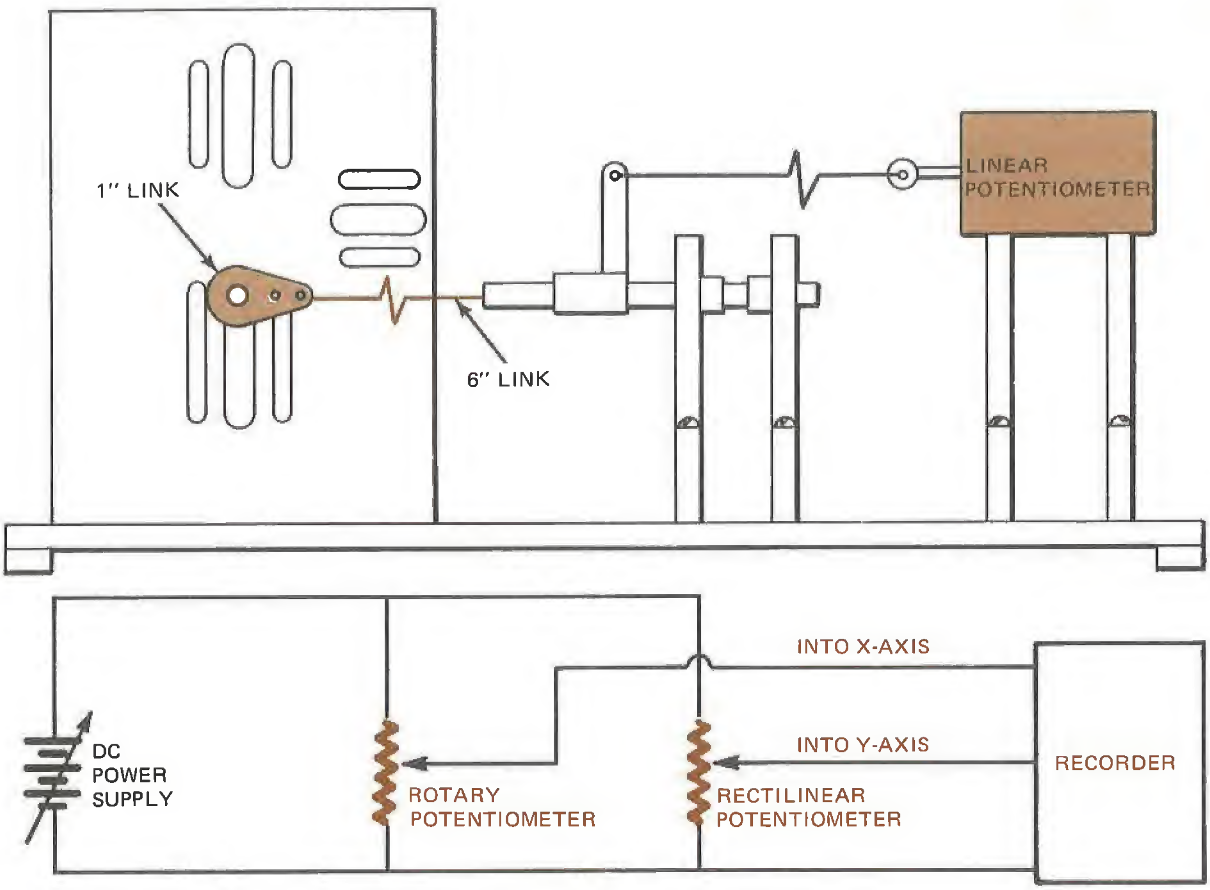


Fig. 6-15 Experimental Equipment (cont'd)

5. Using a velocity polygon, determine the velocity vectors V_{P2} and V_{tP2} of the slider for the angle stated in step 4 for the driver. Assume ω = 50 RPM for the driver.
6. What are the velocity vectors, V_{nP2} and V_{nP1}?
7. With an acceleration polygon, graphically determine the acceleration components, a_{nP2} and a_{tP2}, of the slider for the angle given in step 4.
8. What is the acceleration vector a_{P2} for the slider?
9. Repeat step 5 for a drive link angle of 180° CW from the positive X-axis.
10. Calculate the percent difference between the values of velocity vectors V_{P2} determined by steps 5 and 9.
11. Why should V_{P2} have the value you found in step 9?

ANALYSIS GUIDE. What do you think would be the major sources of error in this experiment? You should consider such things as human error in measurements, loose play in the linkage, and the inability to find the *exact* value of the slope at a point on a curve.

PROBLEMS

1. For the slider-crank linkage shown in figure 6-16, what are the values of V_{P1}, V_{P2}, V_{P1P2}, a_{P1}, a_{P2}, and a_{P1P2}?

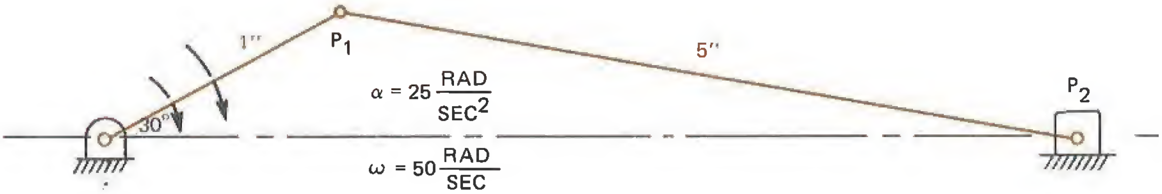


Fig. 6-16 Slider-Crank

2. Given the four-bar linkage in figure 6-17, find the values of a_{P1}, a_{P2}, and a_{P2P1}.

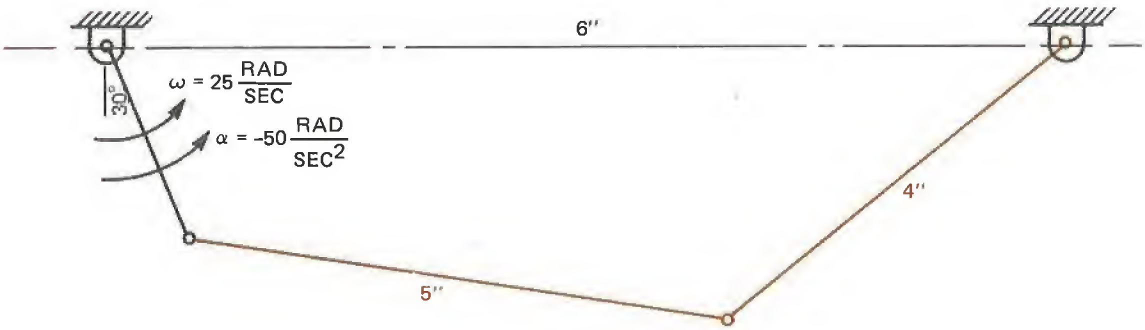


Fig. 6-17 Linkage for Problem 2

3. For the linkage of figure 6-18A, what is the magnitude and direction of the acceleration vector for point P_2?

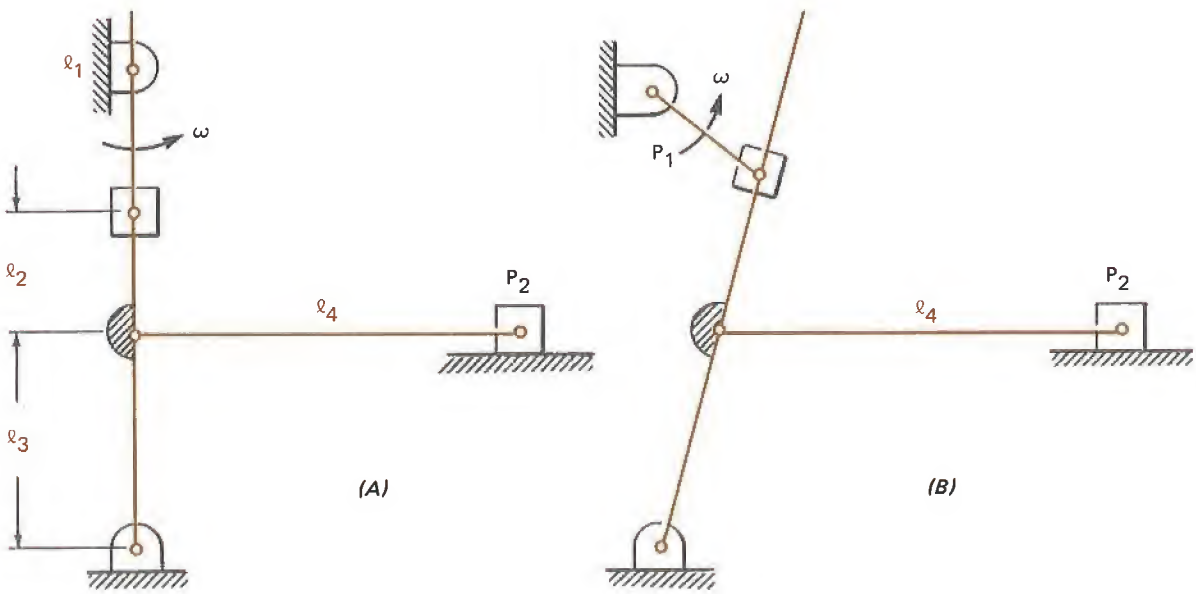


Fig. 6-18 Linkage for Problem 3

4. Repeat problem 1 for a driver link length of 0.75 in. and a connecting rod length of 6 in..

5. In figure 6-18A, for ℓ_1 = 1 in., ℓ_2 = 2 in., draw a sketch of a_{P2} versus ℓ_3, as ℓ_3 is varied from zero to infinity. Show developmental work.

experiment 7 STRAIGHT-LINE MECHANISMS

INTRODUCTION. An interesting device that has many applications is a straight-line mechanism. In this experiment we will investigate some of the characteristics of this linkage.

DISCUSSION. A type of motion required for some industrial applications is straight-line displacement. This is a case where movement of one point of a linkage is along a straight path. Requirements may be such that the mechanism is to have different velocities at different parts of the cycle, or the velocity may need to be constant for part of the cycle.

A common example of a straight-line mechanism which has different velocities for different parts of the curve is that of a shaper mechanism. In this device the return stroke is usually much faster than the cutting stroke.

One way to achieve this type of motion would be to connect a straight-line linkage shown in figure 7-1 to the quick-return linkage in figure 7-2. This combination will yield the linkage shown in figure 7-3 which has straight-line motion with a fast return and could be used in a shaper linkage.

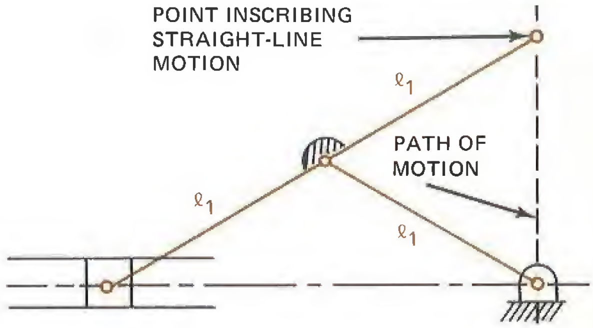


Fig. 7-1 Straight-Line Linkage

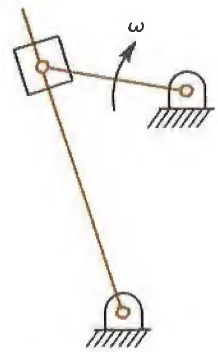


Fig. 7-2 Quick-Return Linkage

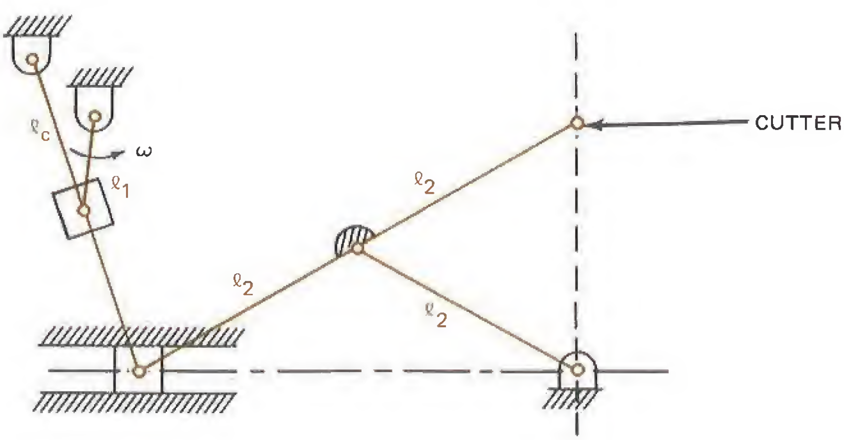


Fig. 7-3 Quick-Return Straight-Line Linkage

There are many other situations in which a straight-line mechanism could be used. Examples of these include marking along a straight line, cutting along a straight line or displacing another link or point along a straight line.

Other mechanisms may also be used to generate straight-line motion as shown by the pair of cylinders in figure 7-4 which generate an infinite number of straight lines. Points P_1 and P_2 are marked as typical examples.

A mechanism that is slightly more complex but gives excellent results is shown in figure 7-5. In this linkage, when either link OP_2 is moved, link $O'P_1$ either pulls or pushes at point P_1, moving point P_3 in or out. With the link lengths as shown, point P_3 will travel in a straight line when link OP_2 is moved.

The linkage in figure 7-5 may also be modified to draw portions of large circles by changing the length of link $O'P_1$ while keeping the length of OO′ constant. If $O'P_1$ is made larger than OO′, then the path traveled by point P_3 will be concave toward point O. If $O'P_1$ is made smaller than OO′, the path will be convex toward point O.

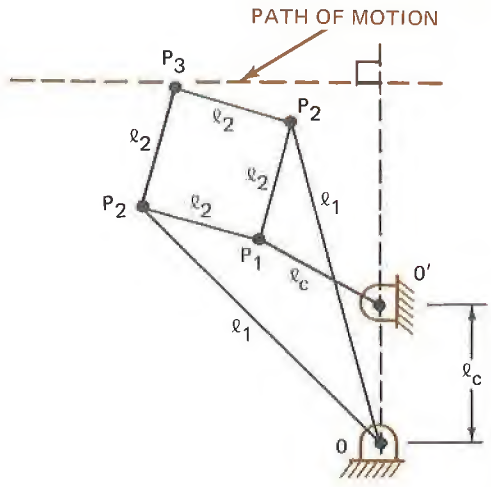


Fig. 7-5 Straight-Line Generator

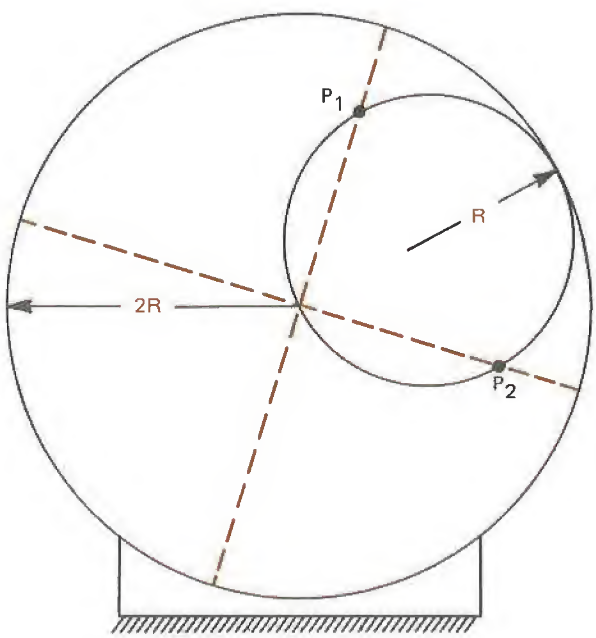


Fig. 7-4 Straight-Line Generator

The linkages in figure 7-4 and 7-5 are unusual in straight-line generators because they move *only* in a straight path for their *full cycle* of operation. Much more common are linkages in which only a *portion* of their path is a straight line or is assumed to be a straight line as in figure 7-6. This linkage

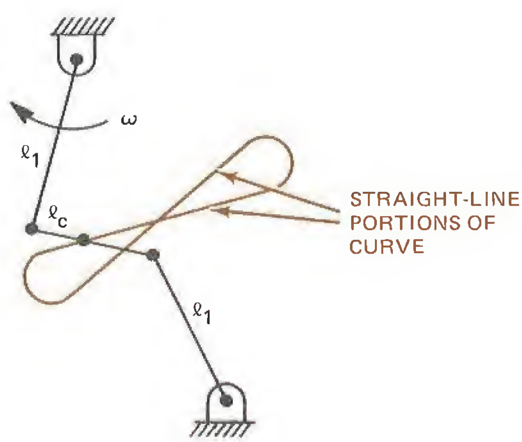


Fig. 7-6 Straight-Line Portions

may be connected to another linkage, however, with movement *limited* to straight-line motion only

Linkages which generate straight-line motion are often used in assembly lines. A possible use could be a bottle-capping function, where the capping mechanism would be at the point describing straight-line motion. Another possibility is a material-cutting device, where the cutting mechanism is again placed at the point having the straight-line motion.

This brings up the problem of having the straight-line point move at a constant velocity over the portion of the straight line where the operation is desired. It is apparent that having the capping or cutting mechanism move in a straight line along the assembly line would not be practical unless the materials were moving at a constant velocity. In the bottle-capping operation, the result of a varying velocity during capping would probably be a stockpile of broken bottles.

Constant velocity for a point on a linkage can be attained by connecting a second linkage to the first and choosing the second linkage to move the first linkage point at a constant velocity over the desired part of the curve.

One procedure for doing this would be to obtain the displacement and velocity curves for the straight-line linkage point. Then use the method of instantaneous centers or velocity polygons repeatedly to find the velocity curve necessary at the point of connection of the second linkage to give the point of straight-line motion a constant velocity over the desired part of the curve.

If, at the point of straight-line motion, the velocity curve is as shown in figure 7-7 and the *desired* velocity is *constant* at a in./sec, then the second velocity curve would be found as illustrated by figure 7-8. Velocity polygons, etc., could then be used to find the required velocity and displacement curves at the point of connection of the secondary linkage.

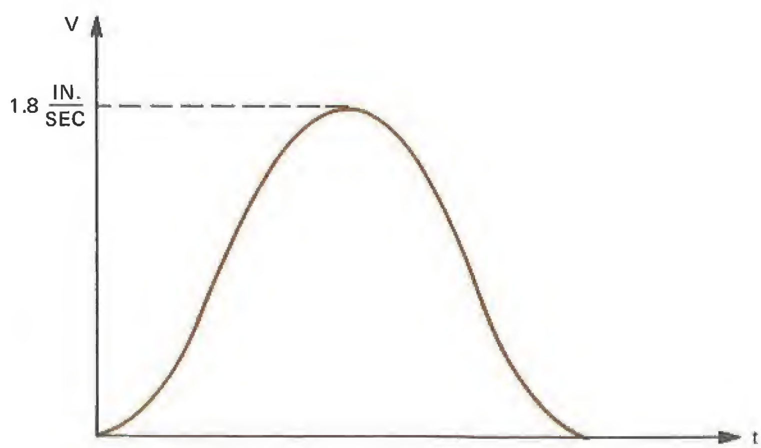


Fig. 7-7 Velocity Curve

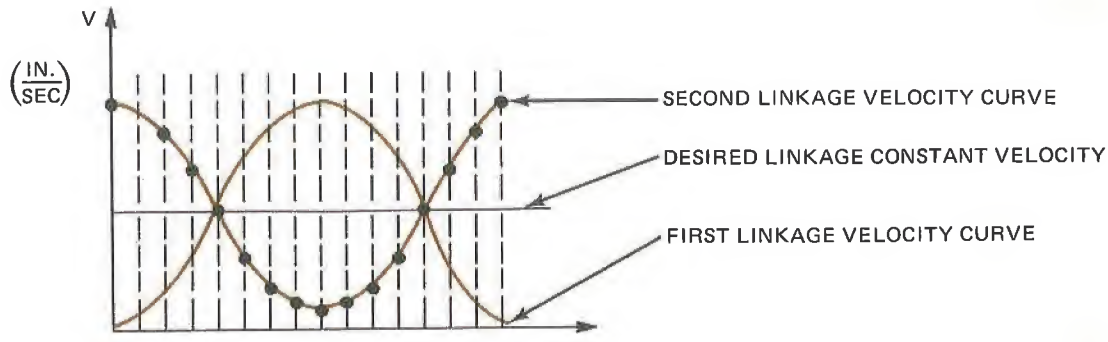


Fig. 7-8 Constant Velocity Curve

We see from figure 7-8 that the second velocity curve is merely the first curve inverted and moved upward, so that the desired value of constant velocity bisects both curves.

The velocity curve for the second linkage can now be graphically integrated to find the displacement curve and a suitable linkage found from a book of linkage displacement curves.

With this procedure a mechanism could be built which would have a straight-line motion at constant velocity for part of the cycle.

In some cases it may be necessary to have a constant acceleration rather than constant velocity over part of the curve. This possibility might arise if measurements were desired to be made on a falling object, and the measurements could not be made instantaneously as the falling object went by the device. The procedure for obtaining constant acceleration would be the same as that for constant velocity, using acceleration curves instead of velocity curves.

A slider-crank is a simple form of a straight-line motion mechanism with the slider providing the straight-line motion. In this linkage, the slider is *forced* to move in the straight path, and the velocity or acceleration can be made constant by the method discussed.

We may also use the linkage in figure 7-6 as a means to provide *dwell* time in an operation. One way to do this would be to use a scotch yoke at the point of connection with the slot parallel to one of the straight-line paths as illustrated in figure 7-9.

Also, since the second straight-line portion of the curve will give the scotch yoke a linear displacement curve, the velocity for that portion will be constant and there will be zero acceleration.

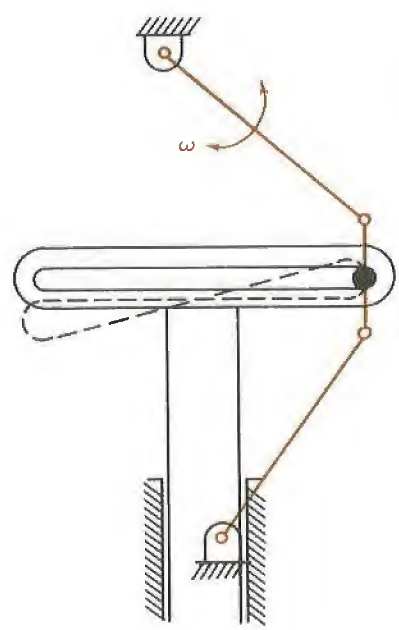


Fig. 7-9 Dwell Type Linkage

MATERIALS

- 2 Bearing holders with bearings
- 2 4 in. X 1/4 in. shafts
- 3 Collars
- 1 Slider
- 1 4 in. link
- 1 2 in. lever arm
- 1 Breadboard with legs and clamps
- 1 Rotary potentiometer
- 1 Linear potentiometer
- 1 X-Y or strip-chart recorder
- 1 DC power supply
- 1 Straightedge
- 1 Engineer's scale
- 1 Divider

PROCEDURE

1. Check all instruments to be sure they are undamaged.
2. Connect the experimental equipment as shown in figure 7-10.

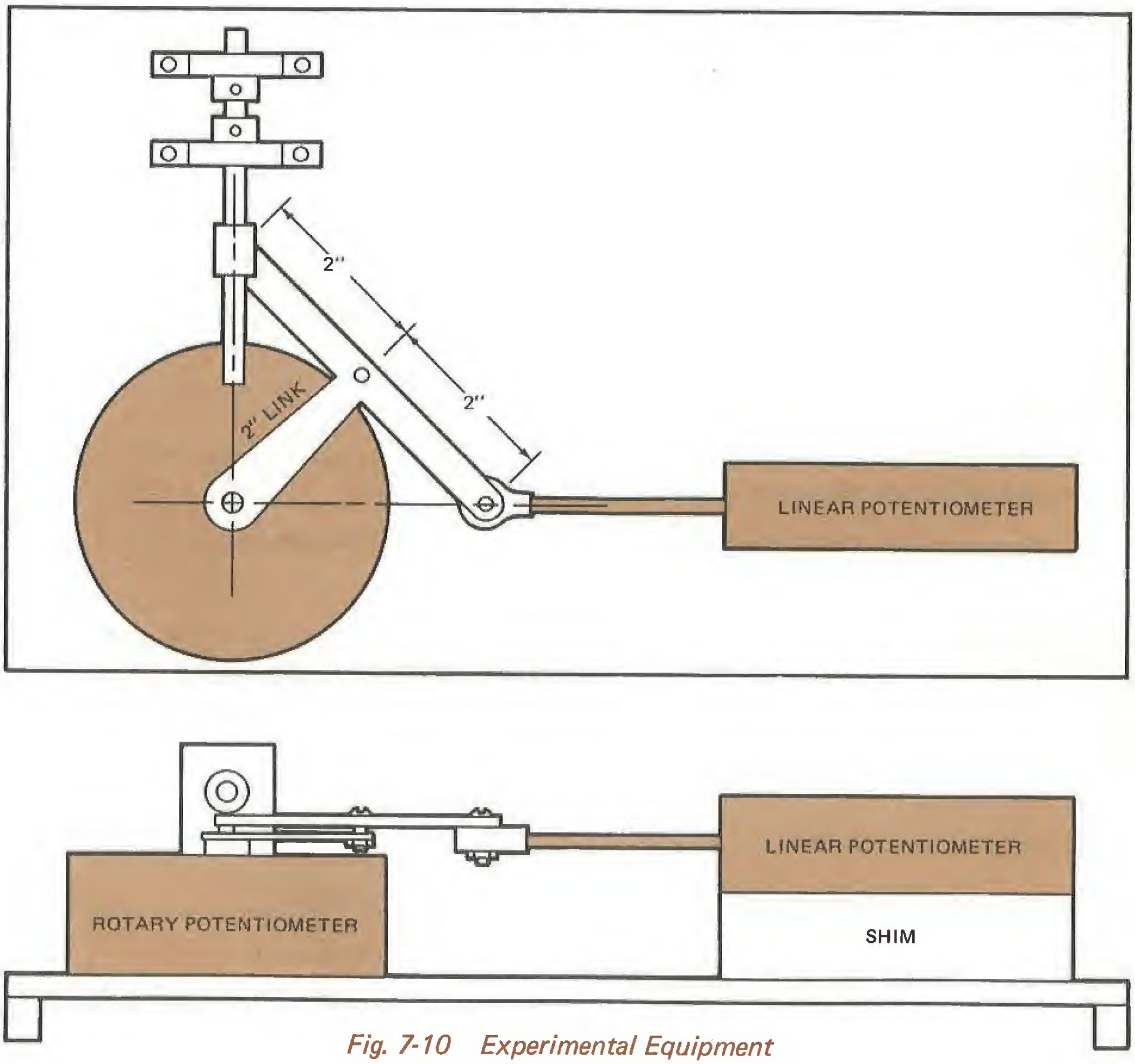
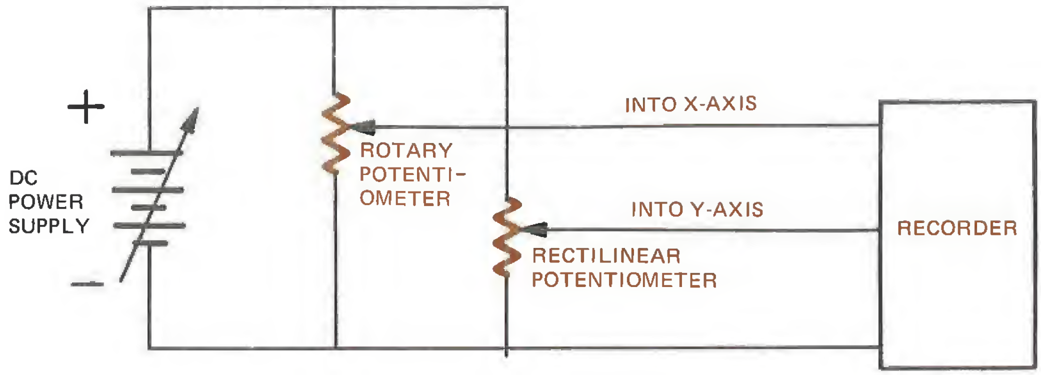


Fig. 7-10 Experimental Equipment



Experiment 7-10 Experimental Equipment (Cont'd)

3. Turn on the equipment, being careful to prevent full-scale deflection of the recorder.
4. Operate the linkage and obtain a curve of the displacement of the connecting link end versus the angle of the rocker link.
5. Graphically differentiate the displacement curve to obtain the velocity curve.
6. Considering *only* the velocity curve from step 5, find the second velocity curve necessary to give an arbitrary, constant value of velocity when the two curves are combined.
7. Graphically differentiate the velocity curve from step 5 to obtain the acceleration curve.
8. Using the procedure from step 6, find the second acceleration curve necessary to give an arbitrary, constant value of acceleration.
9. Disconnect the rectilinear potentiometer from the linkage.
10. Place a pad of paper under the point of straight-line motion, put a pencil in the hole, and operate the linkage to *verify* that the motion is straight-line.
11. Using a straightedge, draw a straight line parallel to the line obtained in step 10.

ANALYSIS GUIDE. What single thing do you think caused the most error in this experiment? What do you think are the major reasons that the line obtained in step 10 was not perfectly straight? You might consider such things as play in the linkage, human error in assembling the linkage, etc.

PROBLEMS

1. Find the part of movement of point A on the linkage of figure 7-11.
2. For the linkage in figure 7-12, find the path of displacement of point A.
3. In figure 7-13, find the motion which points P_1 and P_2 undergo during rotation of the drive link, OP_3, from 0° to 90°.

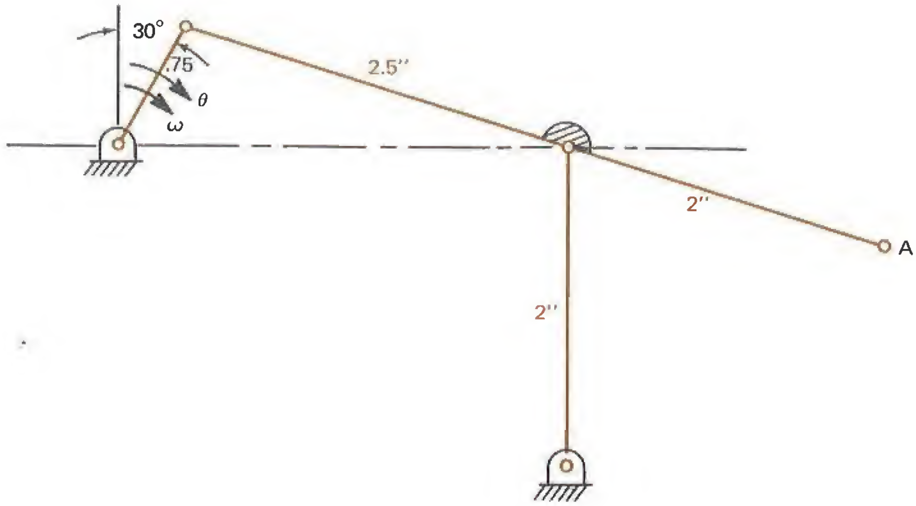


Fig. 7-11 Linkage for Problem 1

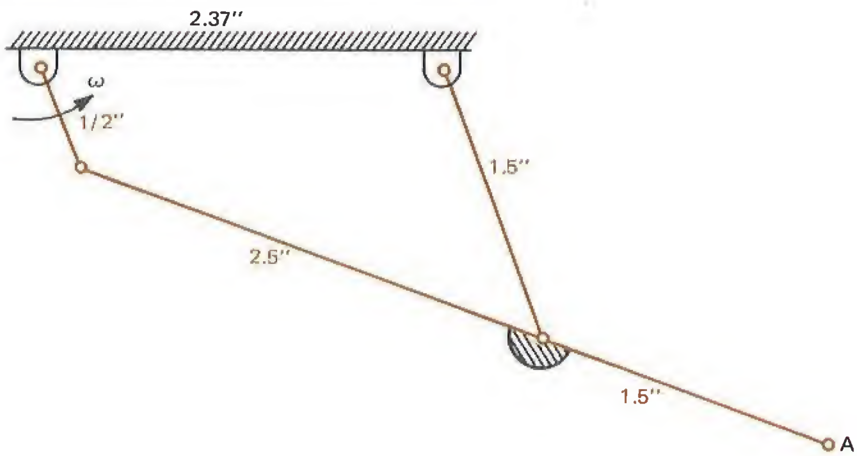


Fig. 7-12 Linkage for Problem 2

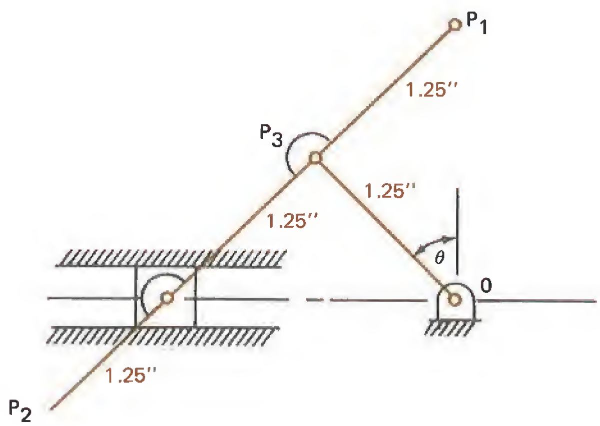


Fig. 7-13 Linkage for Problem 3

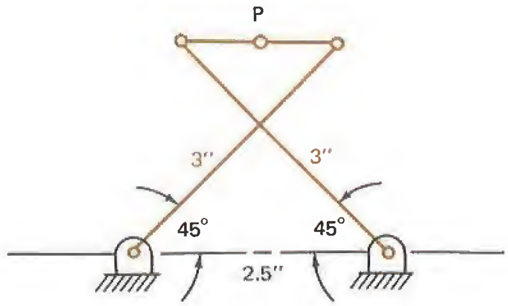


Fig. 7-14 Linkage for Problem 4

4. Draw the path of motion of point P on the linkage of figure 7-14 between limit positions.
5. Draw the path of motion for point P in figure 7-15 for a displacement of 1.5 in..
6. In figure 7-3, what feature of the return stroke would prevent the linkage from being a practical one for a shaper? How could this be corrected?

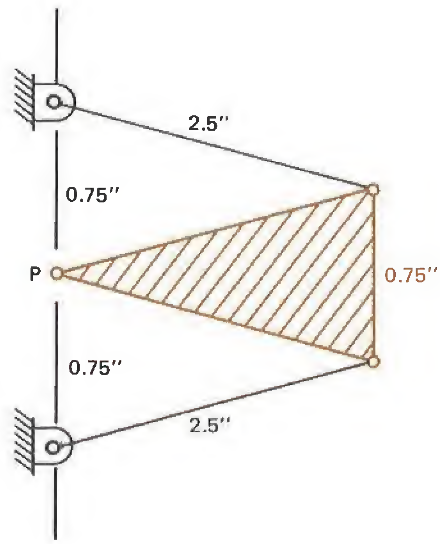


Fig. 7-15 Linkage for Problem 5

experiment **8** *TOGGLE MECHANISMS*

INTRODUCTION. Toggle mechanisms amplify force at the expense of displacement. They have many uses, including rock crushing and hand tool latching. In this experiment, we shall examine some of the characteristics of these mechanisms.

DISCUSSION. Toggle mechanisms have become more important in the past few years than ever before. The advent of flight in outer space, with a large pressure differential between the spacecraft and its surroundings, brought about the need for a *foolproof* locking mechanism on the hatches. In industrial applications, tanks have been developed which will withstand tremendous pressure differentials, and better methods of sealing had to be developed. Punch presses and other tools are developed to a higher degree than in the past. These and other developments have led to the present need for more and better toggle mechanisms to use as force amplifiers, or force amplifiers with a locking position.

There are many applications for which some type of toggle mechanism is desired. Toggle mechanisms are based on a slider-crank linkage for most applications, although a five-bar linkage or several other types of linkages may be used.

These linkages all work in the same manner, with the ratio of input displacement to output displacement approaching infinity at the limit positions. Since work is force times displacement (W = FX), and the work of the input force is equal to the work of the output force, neglecting losses, then

$$\frac{F_{out}}{F_{in}} = \frac{W_{out}/X_{out}}{W_{in}/X_{in}} = \frac{WX_{in}}{WX_{out}} = \frac{X_{in}}{X_{out}} \longrightarrow \infty \tag{8.1}$$

The ratio of forces will reach its maximum value at a limit position, as shown by the slider-crank in figure 8-1.

We see from figure 8-1 that there are two crank positions for which $\frac{F_{out}}{F_{in}}$ approaches infinity. At position 1, the force out is to the right, approaching the maximum value, and at position 2, the force out is to the left, again approaching the maximum value.

We also see from figure 8-1 that, unless the linkage is almost at a limit position, the input force required is much higher. This can

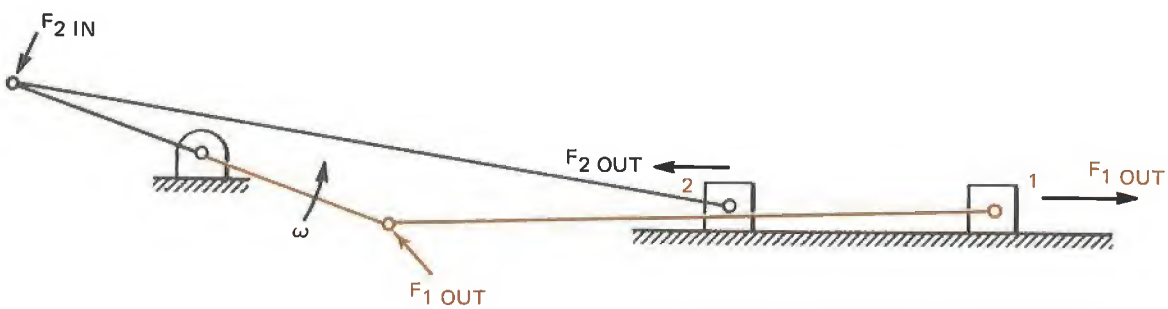


Fig. 8-1 Slider-Crank

Fig. 8-2 Slider-Crank with Handle

be reduced by extending the crank link beyond the point of connection; in effect, forming a handle. This is illustrated in figure 8-2 for a slider-crank.

With the handle added to the drive link, a very large force ratio can be generated by the mechanism, even though it is not at the limit positions. Handles are also added to other linkages to increase the force ratio, as illustrated by the linkage of figure 8-3.

A problem common to figures 8-1, 8-2, and 8-3 is that there is no provision for locking the linkages in place. This locking function may be performed as in figure 8-4 by including an overcenter lock.

Fig. 8-3 Increasing the Force Ratio

Fig. 8-4 Overcenter Lock

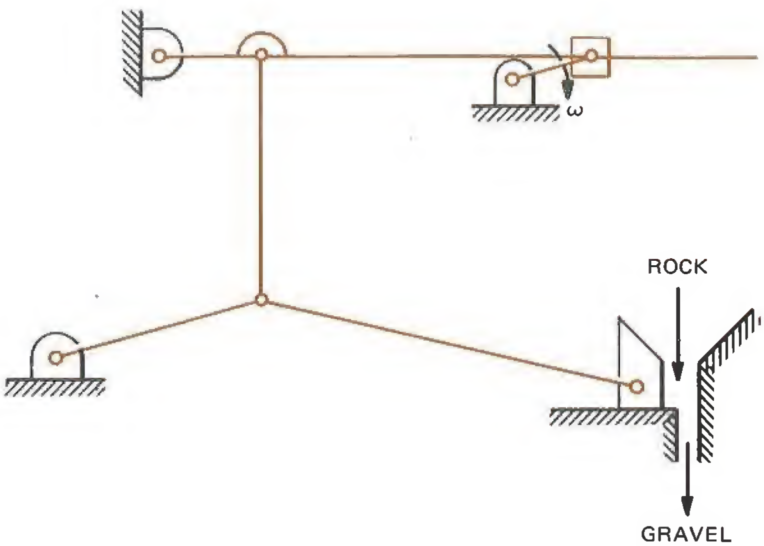


Fig. 8-5 Complete Rock-Crusher

The locking linkage in figure 8-4 has several uses: among them, high-pressure vessel door locks, locking devices on hoists or winches, and, with the addition of a ratcheting mechanism, as a winching device itself.

A toggle mechanism can be combined with a quick-return linkage to form an effective rock crusher, such as that in figure 8-5. The linkage in figure 8-5 is often used with a handle in place of the drive unit, as a pecan shell cracker. There are a great number of configurations in which a toggle mechanism can be drawn, and many different kinds are in service in industry today.

For example, the linkage drive in figure 8-5 could be replaced with that in figure 8-6, which does not have the quick-return feature. The process of replacing one linkage with another is often used, as the equivalent linkage may cost less, be easier to manufacture, or be easier to repair.

If the preliminary design for a pecan shell cracker is as shown in the left side of

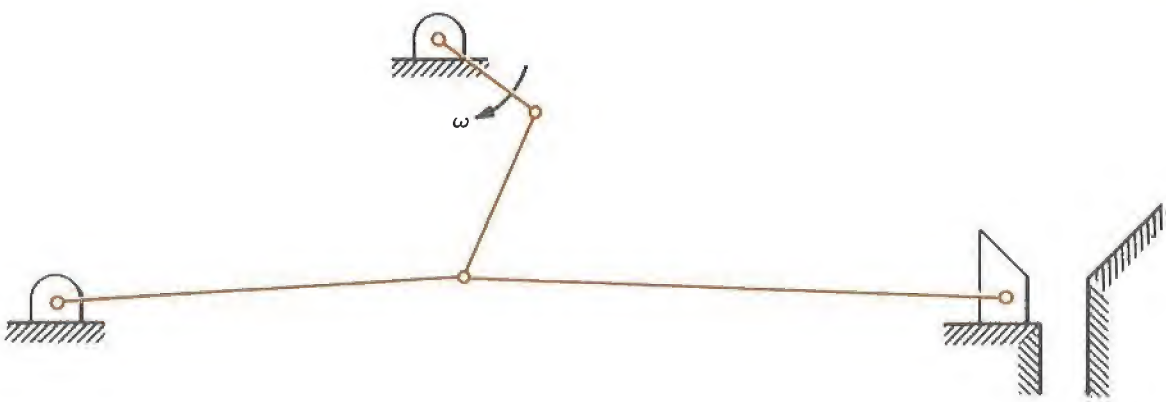


Fig. 8-6 Rock Crusher with Secondary Linkage

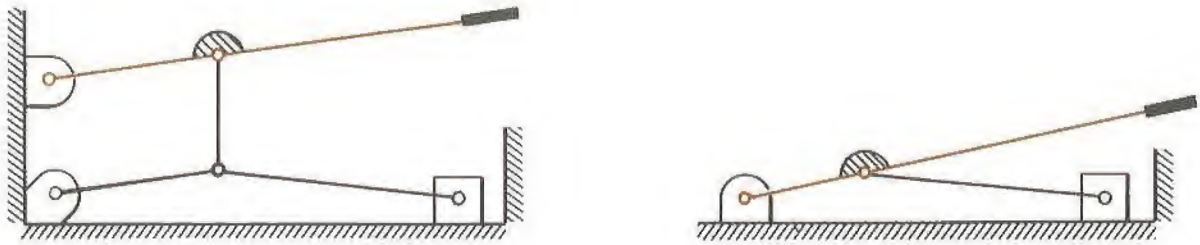


Fig. 8-7 Equivalent Linkage

figure 8-7, the equivalent linkage shown on the right side of the figure would probably be chosen, as it eliminates one right angle in the base plate, two links and two bearings in the linkage.

We need to be aware of this process of replacing a linkage with an equivalent linkage as it may save time, money, and material. The first linkage designed to fulfill a given objective is often far from the best and an equivalent linkage should be substituted for it.

With practice and experience, equivalent linkages are often relatively easy to devise. In toggle mechanisms, only very rarely is the velocity or acceleration considered important, since the desired feature is that of force amplification, and velocities are normally low. Unless there are extremely large masses involved, forces developed by accelerating and decelerating the parts of the linkage are not large enough to cause serious harm. If the acceleration does become large enough to harm the linkage, it may be lowered by increasing the cycle time of the operation, if it is designed to leave the linkage in the same shape. If the cycle time needs to be constant, then the linkage may be modified to give a different displacement curve, and thus a different acceleration curve.

The main points to remember are that toggle mechanisms are used to provide large mechanical advantage from a purely mechanical linkage, and that equivalent linkages can often be used in place of the original linkage.

MATERIALS

- 1 Breadboard with legs and clamps
- 3 Shaft hangers, 2 with bearings
- 1 4 in. x 1/4 in. shaft
- 1 Lever arm, 3 in. long with 1/4 in. bore hub
- 2 3 in. links
- 1 DC power supply
- 1 Rotary potentiometer
- 1 Linear potentiometer
- 1 X-Y or strip-chart recorder
- 1 Engineer's scale
- 1 Straightedge
- 1 Drafting pencil

PROCEDURE

1. Inspect all components to be sure they are undamaged.
2. Connect the linkage and instruments as shown in figure 8-8.

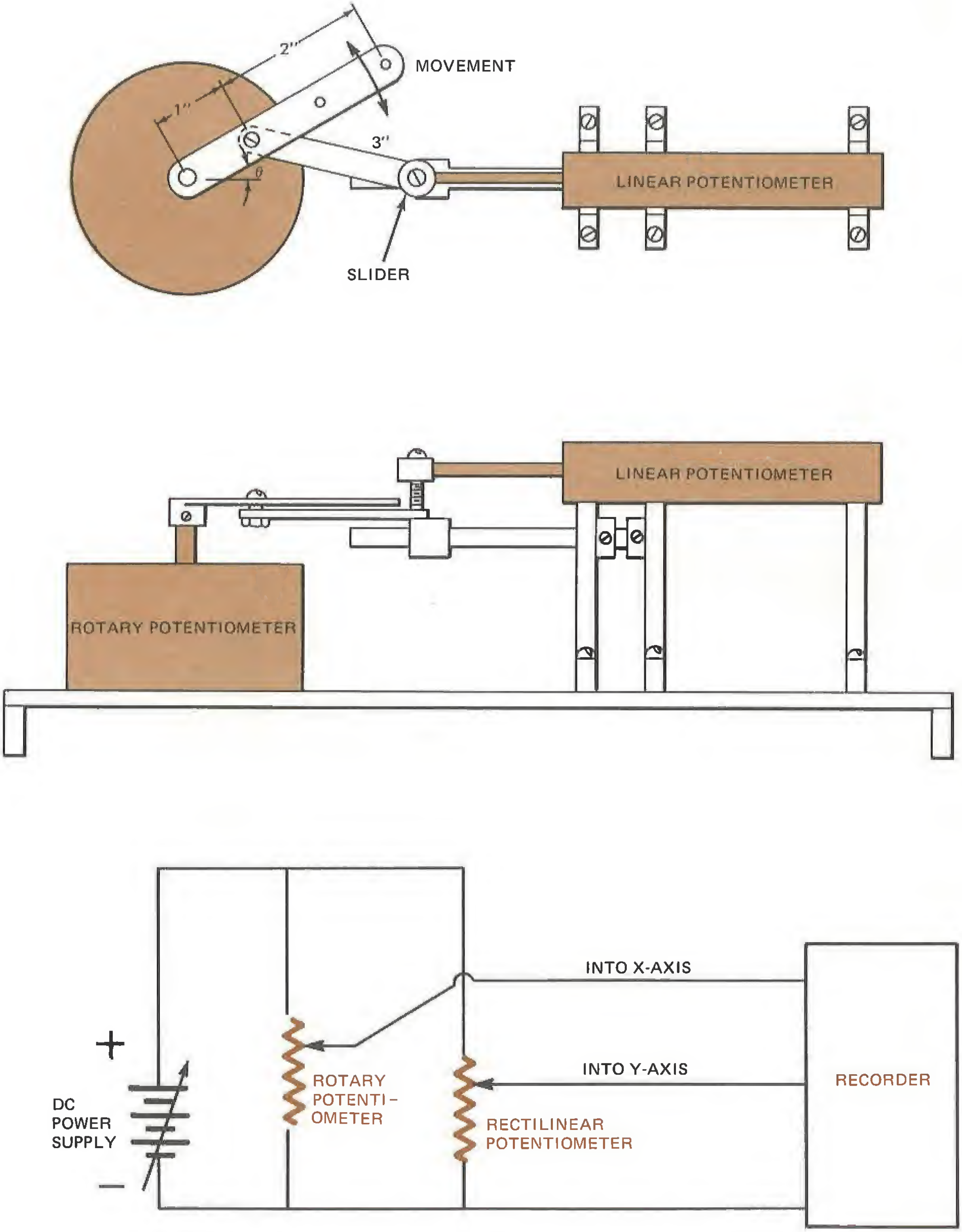


Fig. 8-8 Experimental Equipment

3. Turn the equipment on, being careful to prevent full-scale deflection of the pen.
4. For a crank movement from 90° to 0°, where θ is as shown in figure 8-8, adjust the instrument controls to obtain approximately a 4-in. pen deflection on the Y-axis, and a 6-in. pen displacement along the X-axis.
5. Now, move the crank from 90° to 0° and obtain a displacement curve for the slider.
6. Disconnect the connecting link from the crank link and reconnect it 2 in. away from the crank link center of rotation.
7. Move the crank from 90° to 0° and obtain the displacement plot for this linkage.
8. Disconnect the linear potentiometer from the slider, and reconnect it as shown in figure 8-9.
9. Operate the linkage to obtain a plot of the displacement of the crank end for a 90° increment.

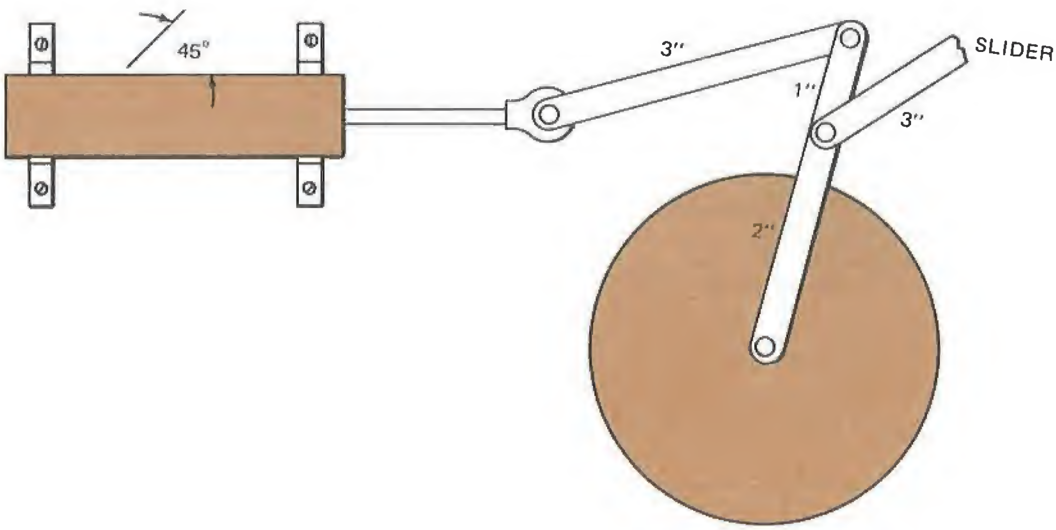


Fig. 8-9 Second Arrangement

10. Graph the *ratio* of crank displacement from step 9 to the slider displacement of step 5, *versus* θ measured by the rotary potentiometer.
11. Repeat step 10, substituting the slider displacement measured in step 7 for that from step 5.
12. Draw the two curves from step 10 and 11 on the same graph coordinates.
13. If we assume that the crank displacement from step 9 is along a straight line, then we can make use of equation 8.1 to consider the plots in step 12 to be those of F_{out}/F_{in} versus θ. Observe visually how the force ratio varies between the two curves of step 12.

ANALYSIS GUIDE. Explain, in your own words, why the curves in step 12 are different. Will the curves asymptotically approach a common value? If so, why? What things do you think will cause the greatest sources of error in this exercise?

PROBLEMS

1. Find the displacement curves for the end of the crank and the slider, versus θ of the crank, figure 8-10, for an angular movement of the crank from 0° to 90° CCW from the positive X-axis.

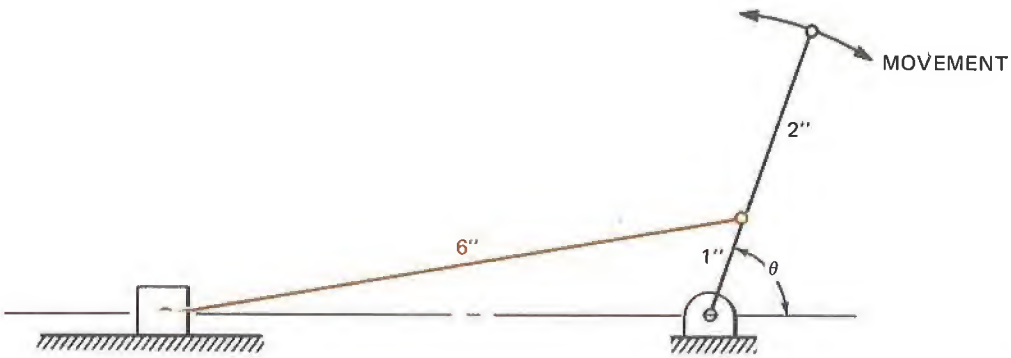


Fig. 8-10 Slider-Crank for Problem 1

2. Using the method of step 12 in the procedure, find the plot of F_{out}/F_{in} for the linkage between θ = 0° and θ = 90°.
3. By inspection of figure 8-10 and the curve from problem 2, sketch the curve for F_{out}/F_{in} between θ = 90° and θ = 180°.
4. Sketch the displacement curve for point P_2 in figure 8-11, for θ between 0° and 180°.

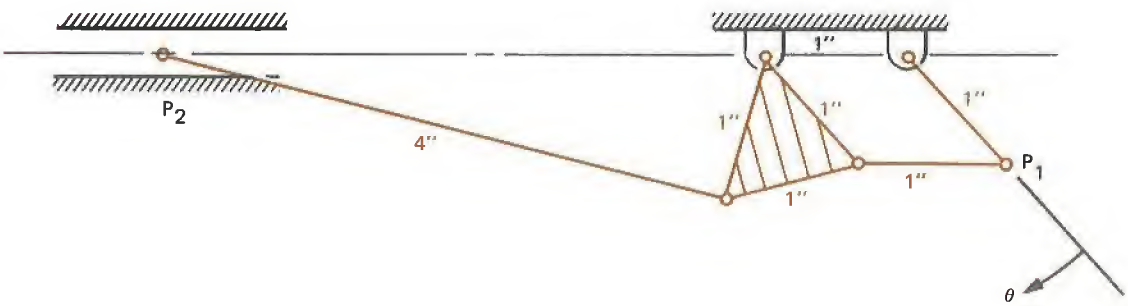


Fig. 8-11 Linkage for Problem 4

5. Find the curve of F_{out}/F_{in} from the displacement plot of problem 4.
6. Would you choose the linkage of figure 8-10 or that of figure 8-11 as the basic linkage for a walnut shell cracker? Why?

experiment 9 MECHANICAL COMPUTING MECHANISMS

INTRODUCTION. Mechanical computing mechanisms can perform multiplication, addition, subtraction, integration, and other mathematical operations. In this experiment we shall investigate some of the ways in which these mechanisms can be used.

DISCUSSION. There are often cases where mechanical computing mechanisms are needed. Among these is the need to find the acceleration of a machine part directly from a velocity meter. If it is not feasible to connect an accelerometer to the part, a mechanical computing mechanism can be connected with a velocity input and an acceleration output. This eliminates the need to graphically differentiate the velocity curve to find the acceleration curve, since the acceleration curve is directly obtained.

Another example would be to directly obtain the velocity curve for the slider of a slider-crank linkage. This could be achieved by giving a displacement input to a mechanical computing mechanism and getting a velocity output.

Both of the preceding examples would require a *differential* computing mechanism.

The procedure above could also be *reversed* by using an *integrating* computing mechanism to integrate the acceleration or velocity input and obtain a velocity or displacement output.

Integration and differentiation operations require fairly complex mechanisms to perform the function. Such mechanisms usually have either a gear train, a linkage, or a combination of the two to integrate or differentiate.

Simpler mathematical operations may be carried out by linkages which are much easier to visualize. For instance, in figure 9-1, the

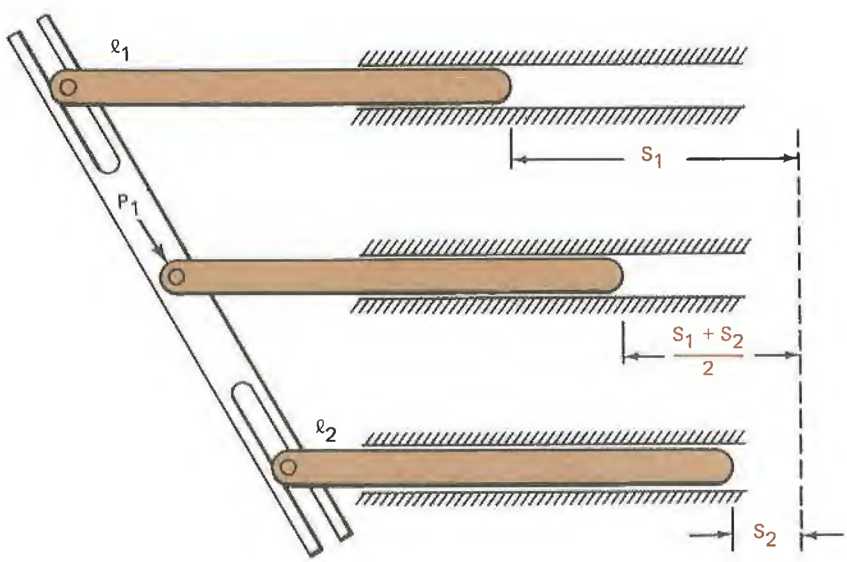


Fig. 9-1 Averaging Mechanism

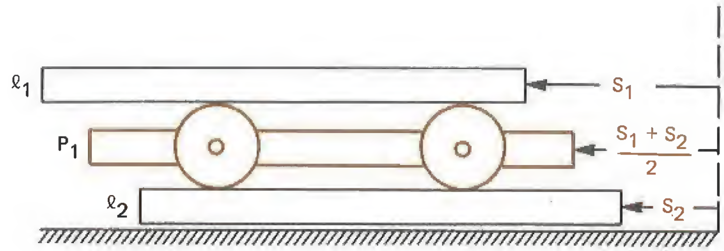


Fig. 9-2 Equivalent Linkage

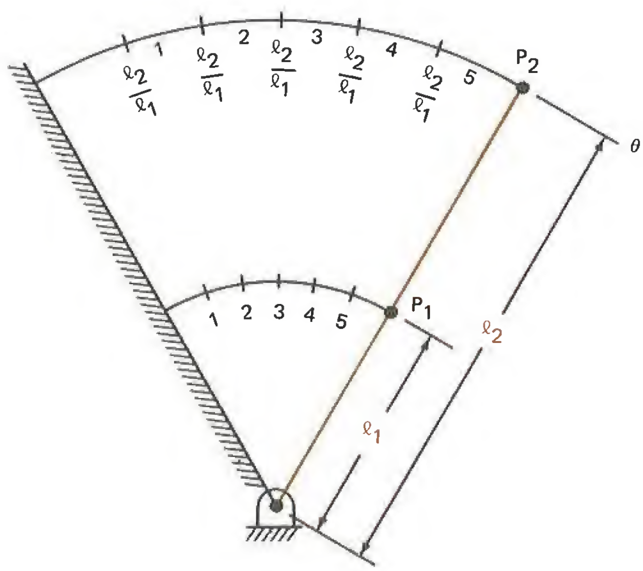
linkage shown will give the average value of displacement of links ℓ_1 and ℓ_2, $(S_1 + S_2)/2$. The value is read from the scale of movement of point P_1.

The linkage of figure 9-1 can be used as an adding mechanism, if we realize that if the average value of displacement $(S_1 + S_2)/2$ is multiplied by 2, the result is $2(S_1 + S_2)/2$, or $(S_1 + S_2)$. This can be read directly from the mechanism *if* the displacement scale of P_1 is *one-half* the scale of S_1 and S_2, resulting in an adding mechanism.

The linkage in figure 9-1 could also be used as a subtracting mechanism. For instance, if we wish to subtract 3 from 8, P_1 is moved to a displacement of 8; ℓ_2 is moved a displacement of 3; and the answer of 5 is read from the displacement of ℓ_1.

Another linkage which will perform the same function as that in figure 9-1 is shown in figure 9-2. This linkage, however, may involve more error than that in figure 9-1 because there may be slippage between the links and the cylinders.

The linkage in figure 9-3 will provide *multiplication by a constant.* We recall that $S = \ell\theta$ and that S varies linearly with distance away from the center of rotation. Therefore, the distance moved by point P_1 can be considered to be multiplied by a constant equal to ℓ_2/ℓ_1, where the multiplied distance is read at point P_2.



S = ℓθ WHERE θ = CONSTANT

$$\theta = \frac{S_{P_1}}{\ell_1} = \frac{S_{P_2}}{\ell_2}$$

$$S_{P_2} = S_{P_1}\frac{\ell_2}{\ell_1}$$

Fig. 9-3 Multiplication Mechanism

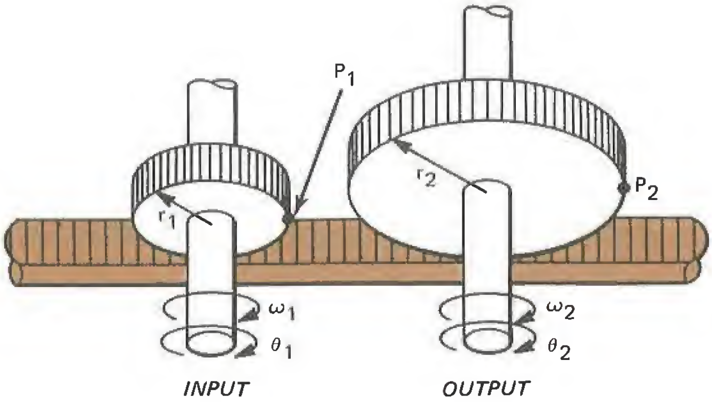


Fig. 9-4 Multiplication Mechanism

The linkage in figure 9-3 may also be used for *division by a constant* by moving P_2 to a value on its scale and reading the value of $\frac{Sp_2}{\ell_2/\ell_1}$ indicated by point P_1. The relationship used in this is

$$\boxed{Sp_1 = \frac{Sp_2}{\ell_2/\ell_1}}$$

As in the previous example, the scale for P_2 may be changed by some scale factor to obtain a different constant of multiplication or division, or ℓ_1 or ℓ_2 may also be changed.

The process of multiplication or division can also be achieved by use of a gear train as shown by the example in figure 9-4.

We observe from figure 9-4 that the constant of multiplication may be varied by changing the radius of either of the gears. If the radius of the second gear is twice that of the input gear, the constant of multiplication is 0.5. If the radius of the second gear is one-half that of the input gear, the constant of multiplication is 2. By reversing the input and output, the constants of multiplication become 2 and 0.5, respectively. As before, the constants of multiplication can also be changed by changing the scale factor on either P_1 or P_2.

Trigonometric functions can be readily generated by several types of linkages. One of the simpler ones capable of generating a sinusoidal output, given a constant angular velocity input, is shown in figure 9-5.

The linkage in figure 9-5 is capable of generating *either* a sine or cosine wave. A sine

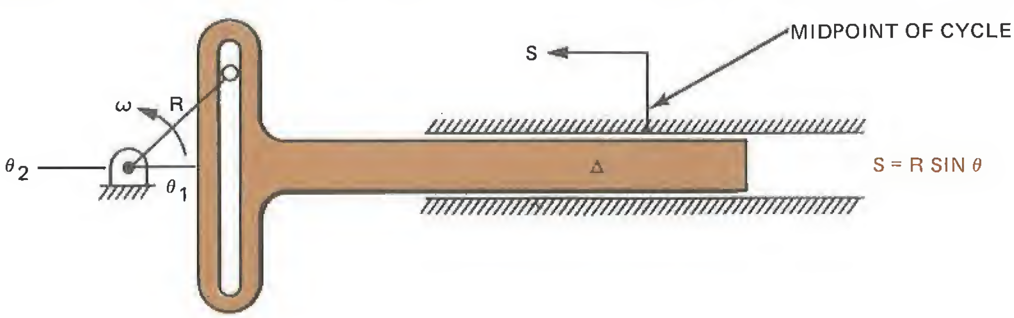


Fig. 9-5 Sinusoidal Computing Mechanism

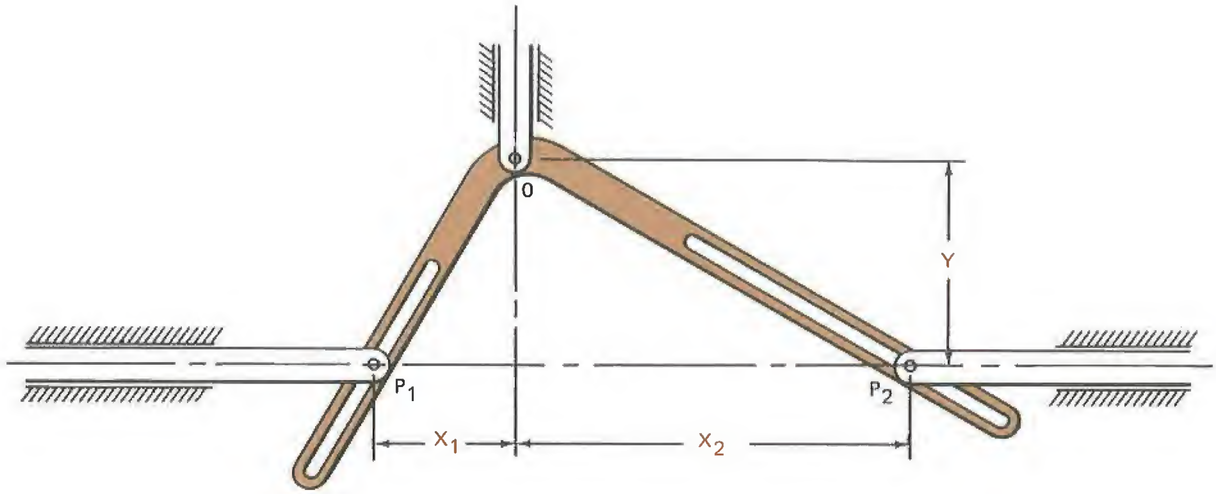


Fig. 9-6 Product Computing Linkage

wave is formed if θ is assumed to be zero at θ_1 in the figure. A cosine wave is formed if θ is assumed to be zero at θ_2 in the figure.

In some cases it may be desired to have a readout of *both* the sine and cosine form of the output at the same time. There are two apparent ways to do this. The first is to connect *two* scotch yokes to the slide instead of one, and put the corresponding scale on each of them, zeroed at different points. An easier method is to place *two* scales along the single scotch yoke's movement in figure 9-5, one each for the sine and cosine output.

A bell-crank mechanism can be used to find the *product of two numbers.* The basic arrangement is shown in figure 9-6.

We observe that if the arms OP_1 and OP_2 are placed at angles of 45° CW and CCW from the negative Y-axis and fixed on the sliding link at point O, then the function that can be found is the square root of a number. This can be proved by similar triangles, since $P_1OP_2P_1$ forms a right triangle.

$$\frac{Y}{X_2} = \frac{X_1}{Y},$$

where $X_1 = X_2$. Then,

$$\frac{Y}{X} = \frac{X}{Y}$$

$$Y^2 = X^2$$

$$Y = X$$

This is a trivial case since it solves the equation $X = \sqrt{X^2} = X$.

The case where P_1 is fixed at a value of unity for X_1 can be shown to solve the equation

$$Y = \sqrt{X_2}$$

As in the preceding example, we have

$$\frac{Y}{X_2} = \frac{X_1}{Y}$$

$$Y^2 = X_1X_2$$

$$\boxed{Y = \sqrt{X_1X_2}} \qquad \textbf{(9.1)}$$

But we let $X_1 = 1$; therefore,

$$\boxed{Y = \sqrt{X_2}} \qquad \textbf{(9.2)}$$

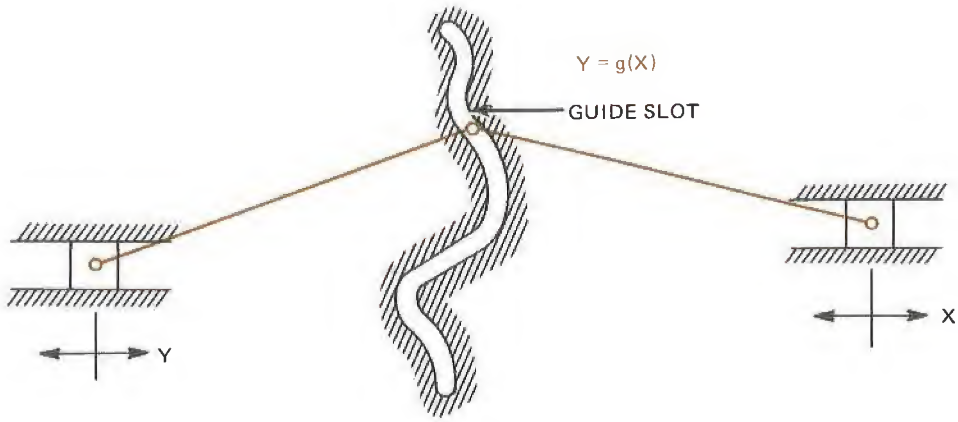


Fig. 9-7 Complex Function Computing Mechanism

The most general case for this linkage would be that described by equation 9.1 for arbitrary, unfixed motion of points O, P_1, and P_2. This would be the square root, Y, of any two numbers, X_1 and X_2.

Functions with one variable may be solved by a linkage such as that in figure 9-7. This is normally used to solve fairly complex equations where the relationship is given by

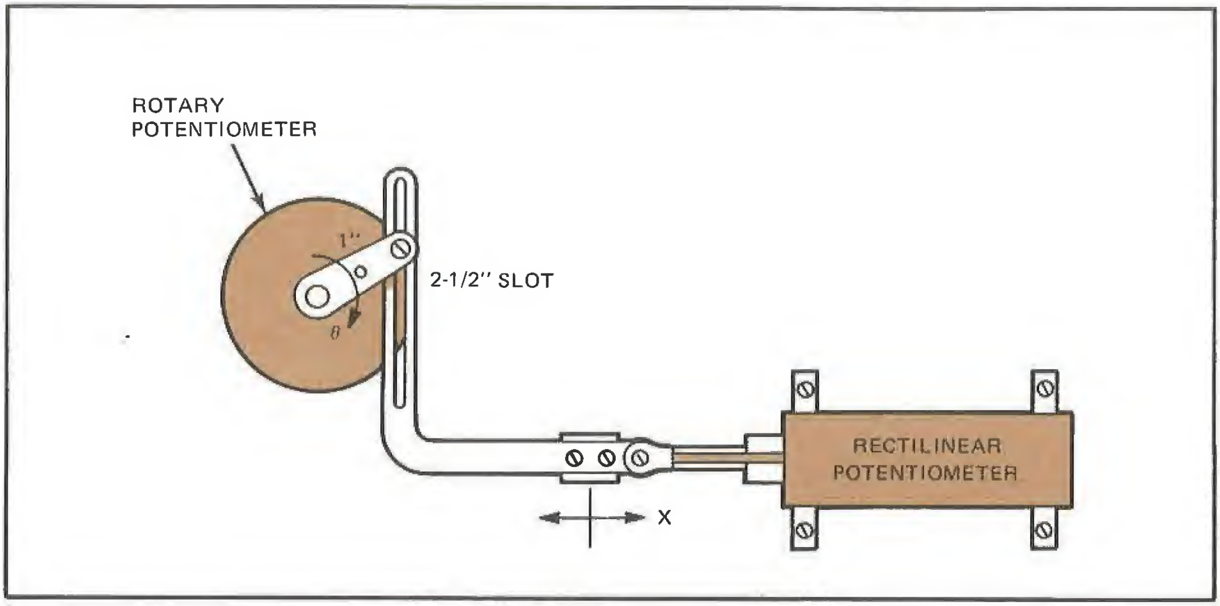
$$\boxed{Y = g(X)}$$

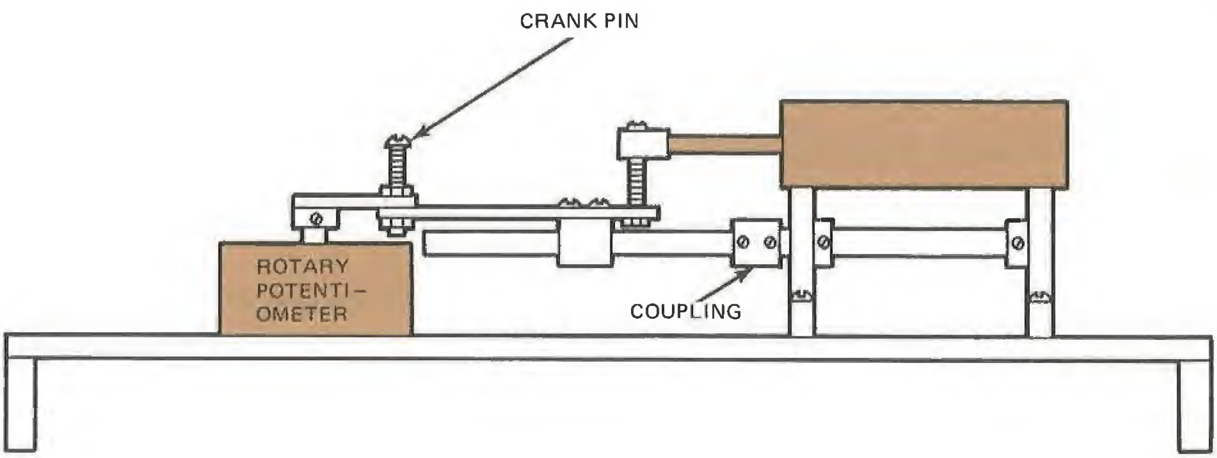
MATERIALS

- 1 2-1/2 in. slotted bell crank
- 1 1 in. link
- 1 Breadboard with legs and clamps
- 2 4 in. x 1/4 in. shaft with 1 slider
- 2 Bearing hangers with bearings
- 1 Rotary potentiometer
- 1 Rectilinear potentiometer
- 1 DC power supply
- 1 X-Y or strip-chart recorder
- 1 Coupling
- 2 Collars

PROCEDURE

1. Check all instruments and components to be sure they are undamaged.
2. Connect the equipment as shown in figure 9-8.
3. Turn the equipment on, being careful not to permit full-scale deflection of the recorder pen.
4. Adjust the controls to obtain a 3-inch high by 6-inch wide plot for one cycle.
5. Turn the crank through 360°, orienting the linkage to obtain one-half of a sine wave.
6. Turn the crank through 360°, orienting the linkage to obtain half of a cosine wave.





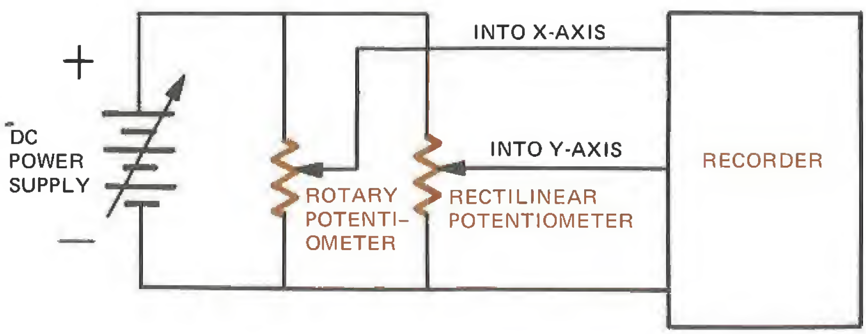


Fig. 9-8 Experimental Equipment

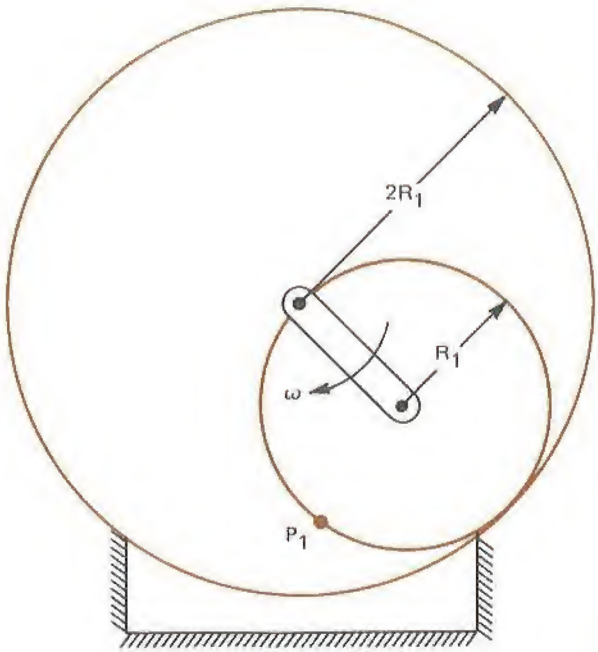


Fig. 9-9 Rolling Cylinders

7. Disconnect the bell crank from the 1-inch drive link and reconnect it 1/2 inch away from the center of rotation of the drive link.
8. Repeat steps 4, 5, and 6.
9. Plot the sine waves from steps 5 and 8 on the same graph and observe visually how they vary.
10. Plot the cosine waves from steps 6 and 8 on the same graph and observe how the two curves are different.

ANALYSIS GUIDE. In your own words, explain *how* and *why* the curves in steps 9 and 10 are different. You should consider human errors, linkage errors, and recording errors.

PROBLEMS

1. What simple change could you make to the time base of the wave forms in steps 5 and 6 to force them to become identical curves?
2. In figure 9-9, if you connected a slider to point P_1 normal to the base, what motion would the slider undergo, assuming there is no slip between the two cylinders?
3. What is the constant of multiplication for the linkage of figure 9-10 with a scale for P_2 0.75 times that for P_1?
4. In figure 9-9, sketch the motion performed by points on the small cylinder at distances from the center of 1/2 R_1.

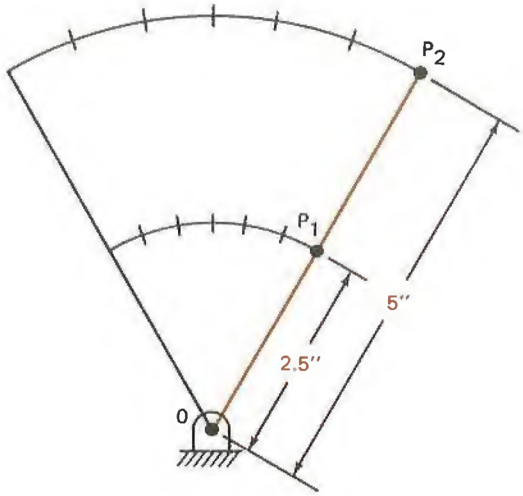


Fig. 9-10 Linkage for Problem 3

5. In figure 9-11, when P_2 is moved one unit, what is the displacement of P_2? (Make any *reasonable* simplifying assumptions required.)

6. What is the generalized relationship between X and Y in figure 9-11?

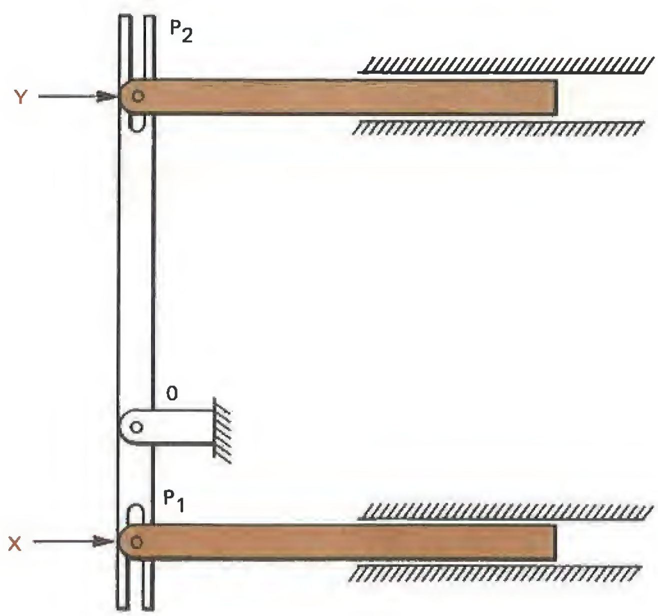


Fig. 9-11 Computing Mechanism

experiment 10 CAM WITH SINGLE FOLLOWER

INTRODUCTION. Cams are direct-contact mechanisms made such that the motion of the cam imparts a specified motion to the follower. In this experiment we will check the characteristic curves of a cam with a single follower.

DISCUSSION. There are three major types of cams normally encountered in industrial applications: *disc, cylindrical,* and *translational* cams. Cams are often used because we can impart almost any desired motion to a follower by using the appropriate cam. Examples of cams in wide-spread usage are the disc cams in internal combustion engines, with all three major types used in machine tools and computers.

In *designing* cams the usual approach is to find the shape of the cam from the displacement curve of the follower. The procedure for a disc cam is to draw the displacement curve, then draw the base circle of the cam to the left of it. The angles from the curve are marked on the cam, then the displacement of the follower at the corresponding angle is transferred to the vertical diameter line of the cam. A circular arc is then used to transfer the displacement to the corresponding angle on the cam and the point is marked. This is repeated for the full 360° circle and the points are connected with a smooth curve. The number of angles marked depends on the accuracy desired. This procedure is illustrated in figure 10-1 for a sinusoidal displacement curve.

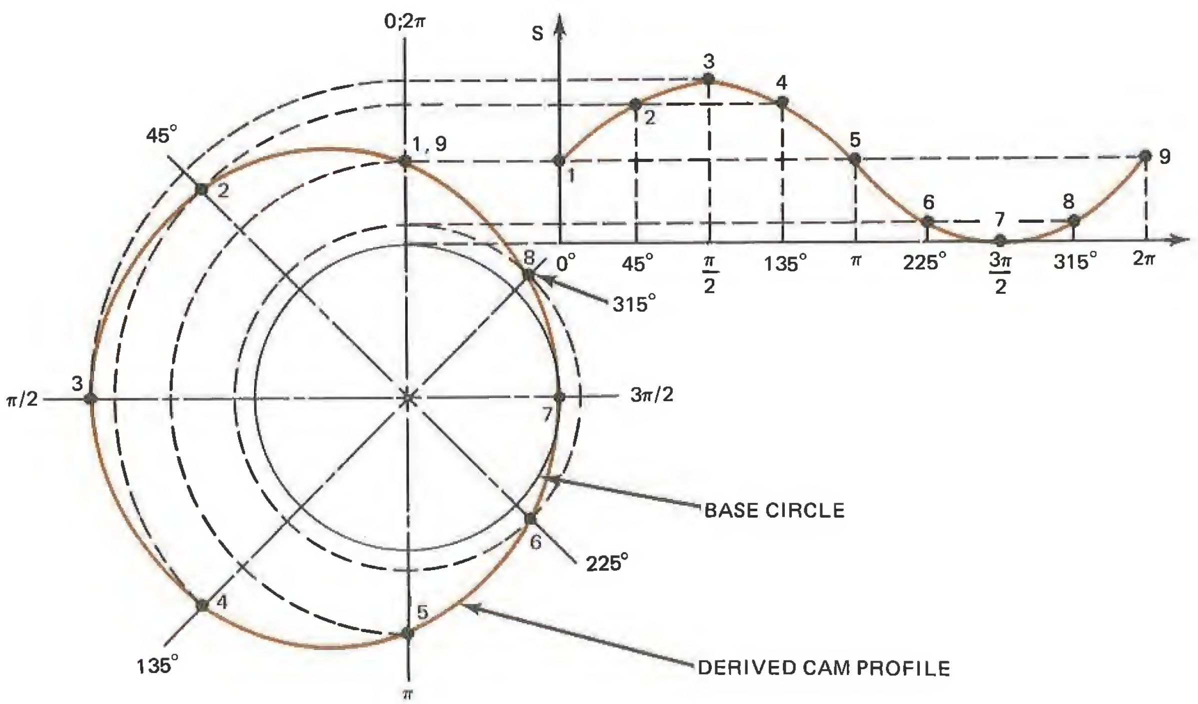


Fig. 10-1 Finding Disc Cam Profile

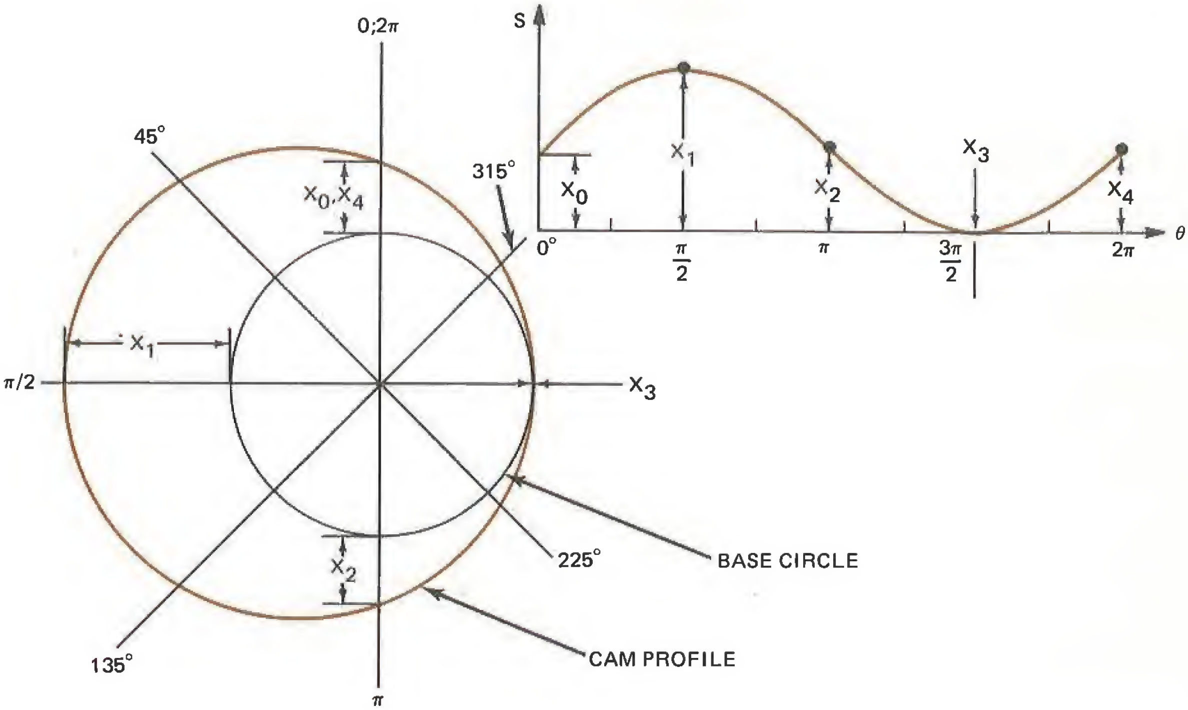


Fig. 10-2 Cam Profile With Calipers

The procedure shown in figure 10-1 may also be reversed, and the displacement curve found for a particular cam. It is apparent that the displacement must be *equal* at 0° and 360°.

A procedure which uses calipers and does not involve so much drawing, *but* should not be attempted until a thorough understanding of finding cam profiles is gained, is given by figure 10-2. This method is faster, but is not as apparent as that of figure 10-1.

The procedures in figures 10-1 and 10-2 may also be used to find the profile or displacement curve for the *positive motion cam* shown in figure 10-3.

The positive-displacement cam shown in figure 10-3 is normally restricted to light usage or small angular velocities. This restriction is due to the fact that when the velocity changes signs (i.e., displacement passes a peak), the angular displacement of the follower *roller* reverses directions. It is apparent that if this cam is used continuously, or at relatively large angular velocities, the follower roller will have a large rate of wear.

This cam, however, has an apparent advantage over the simple disc cam. When there are *space limitations* imposed on the follower and guide, the disc-type cam usually requires a spring to hold the follower in contact with the cam face while the positive-displacement cam does not.

We often find a use for *cylindrical* cams when the follower motion needs to be *parallel* to the axis of the cam. If we used a disc cam, a secondary *linkage* would have to be designed to impart the desired direction of

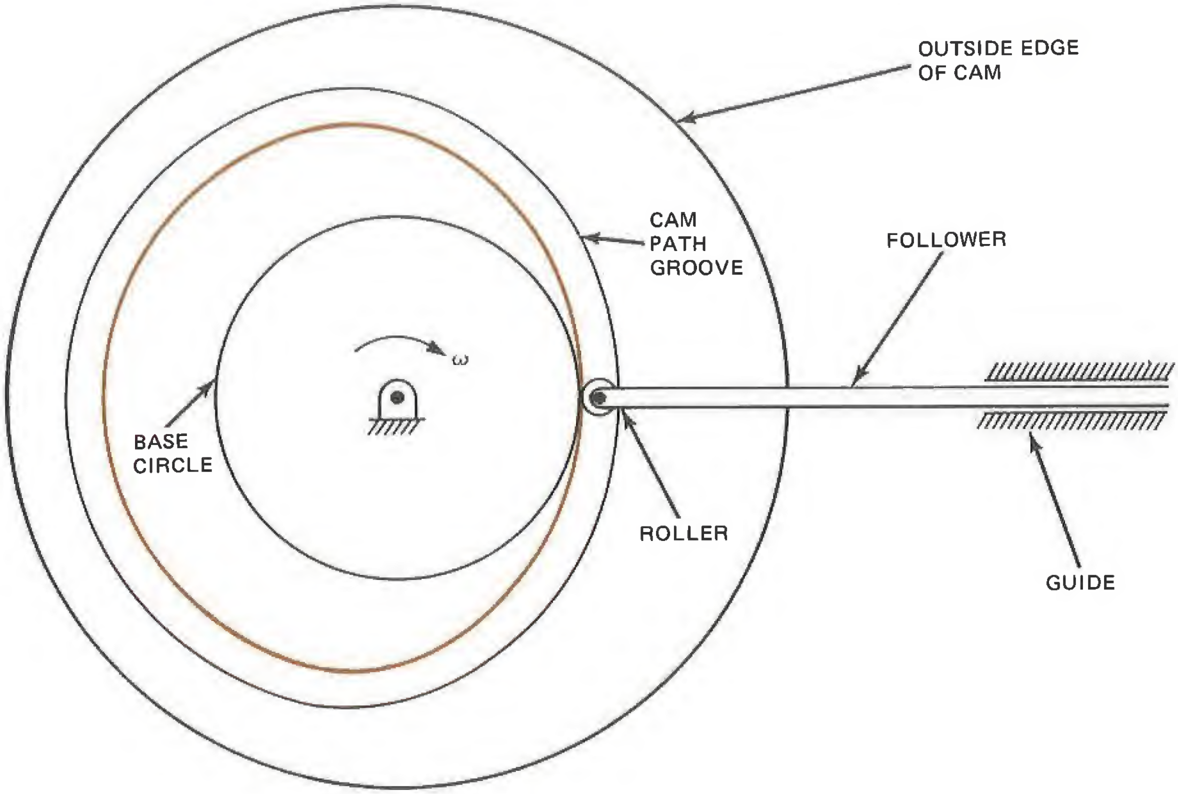


Fig. 10-3 Positive-Displacement Cam

motion to the cam. However, a cylindrical cam may be used to give the follower the desired motion *directly,* as shown by figure 10-4. The follower may also be placed parallel to the axis of the cam, giving the displacement curve of the follower the same

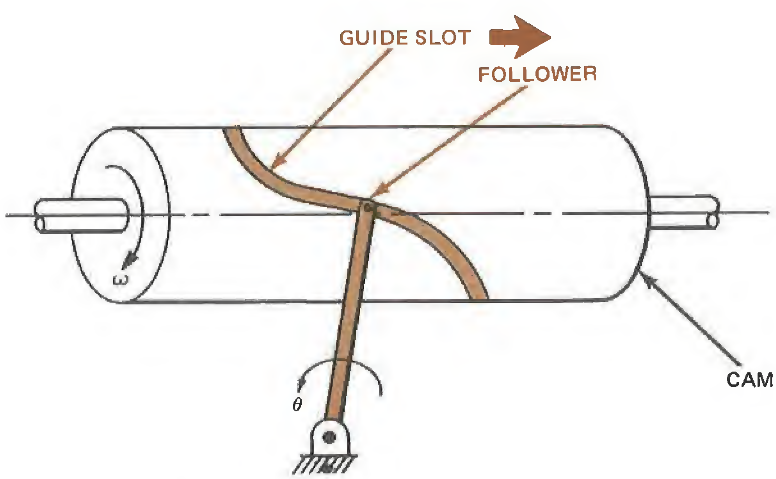


Fig. 10-4 Cylindrical Cam

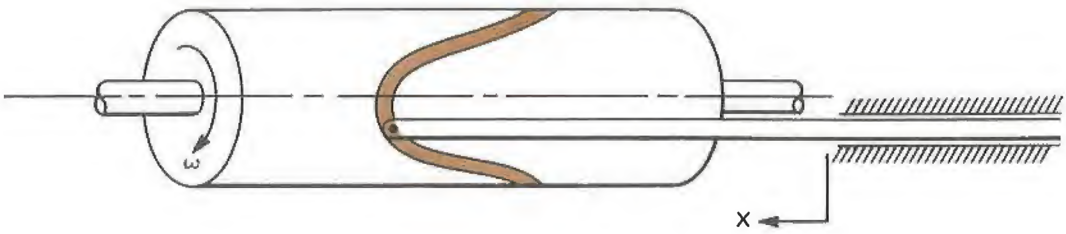


Fig. 10-5 Identical Displacement Follower

value as the displacement curve of the cam. This mechanism is shown in figure 10-5.

Translational cams are often used when a *reciprocating* input is given, and a *perpendicular* output is desired. The translational type of cam is merely a thick plate with one edge in the shape of the follower displacement curve. A typical example of this cam is shown in figure 10-6, with a sinusoidal displacement curve.

In the examples we have discussed so far, we have assumed that the cams move with *constant* angular or rectilinear velocity. This is often not the case, particularly with the translational cam, which may be driven with a crank as in figure 10-7, giving simple harmonic motion (sinusoidal) to the cam.

In this situation, the known parameters are usually the follower displacement curve, and the sinusoidal input with an unknown profile of the cam. A solution can be obtained by plotting the values of input displacement versus follower displacement at

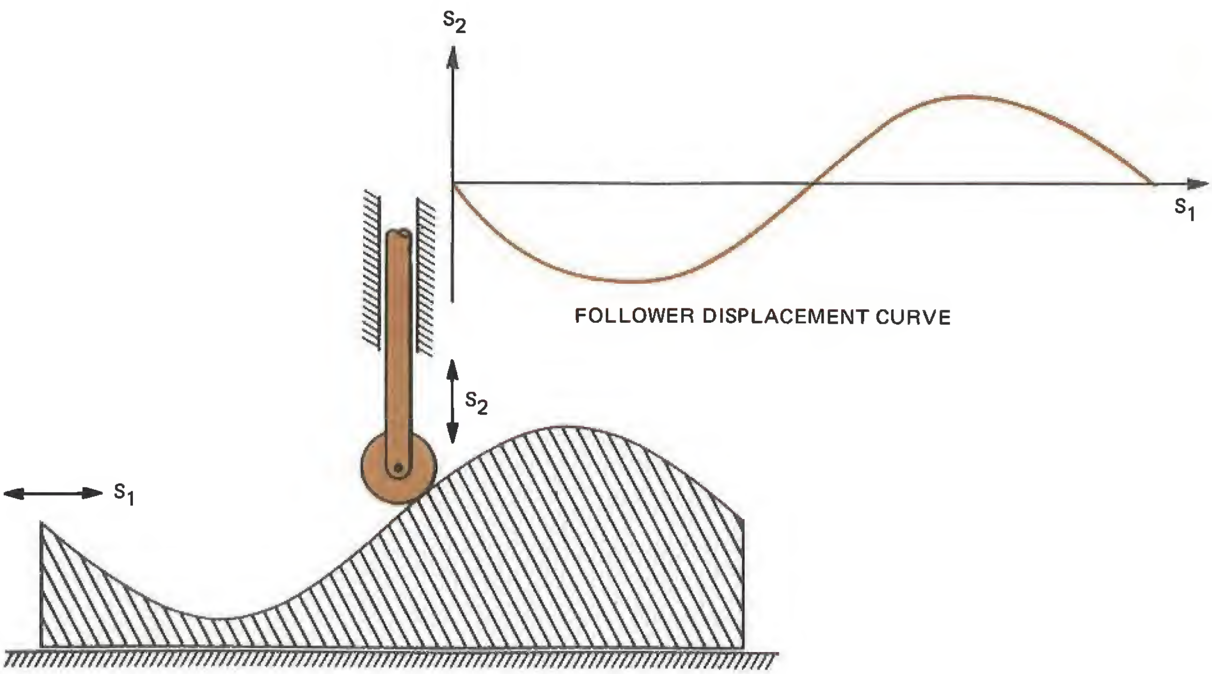


Fig. 10-6 Translational Cam

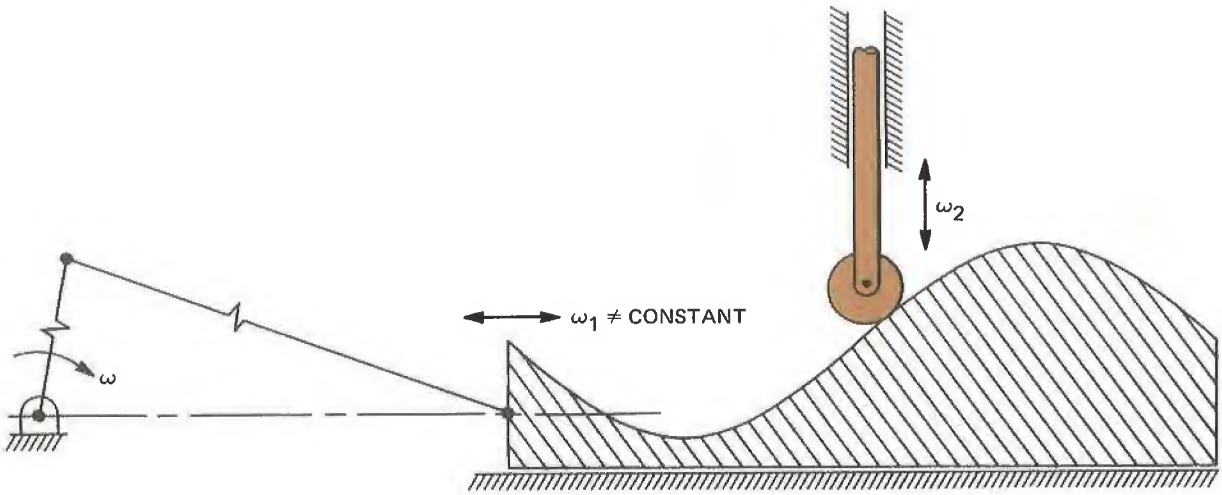


Fig. 10-7 Sinusoidal Input

the same time increments. This will yield the curve of cam height versus cam displacement for part of the cycle as illustrated by figure 10-8.

This method can be used to find any one of the three displacement curves, input, cam profile, or output, for the translational cam when the other two are known.

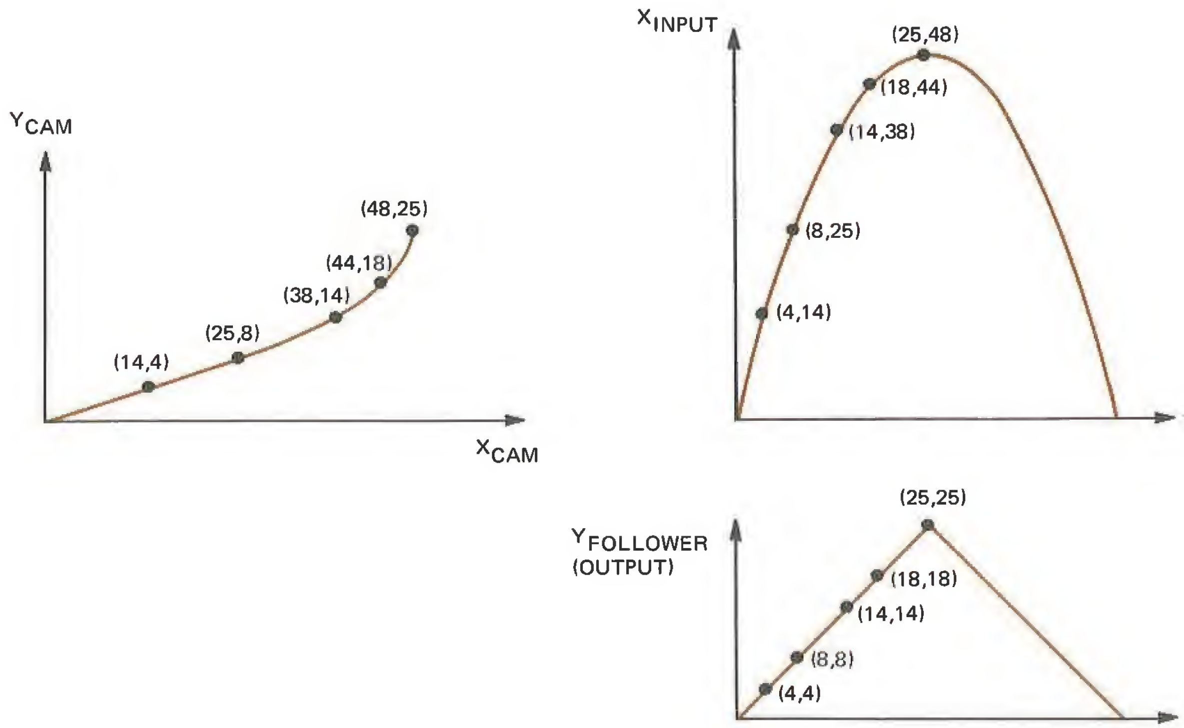


Fig. 10-8 Translational Cam Profile

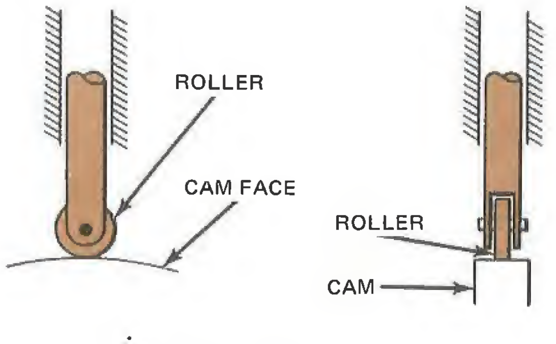


Fig. 10-9 Roller Follower

The followers shown so far have all been of the roller type. There are three types of *followers* in wide-spread use: the *roller, flat-face,* and *knife-edge.* The *roller* consists of a bearing-mounted disc connected to the follower as shown in figure 10-9. Roller-type followers are normally used on disc and translational cams, when the change in curvature of the cam is not too large for positive displacement, and on cylindrical cams. However, with a *small* roller, fairly small cams with *large* changes in curvature are practical.

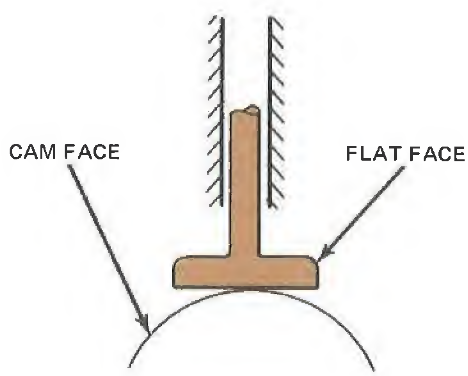


Fig. 10-10 Flat-Face Follower

Flat-face followers have a flat surface which is in contact with the face of the cam with the flat surface usually perpendicular to the axis of the follower rod. This type of follower is shown in figure 10-10. This follower is normally used for disc and translational cams with a fairly small radius of curvature.

Knife-edge followers are built, as the name implies, in the form of a follower rod with a sharp point. Figure 10-11 illustrates a knife-edge follower.

The use of this type of follower should be avoided, whenever possible, as it may be used only with small pressure angles (angle between normal to cam and follower direction of displacement) and has a high wear rate.

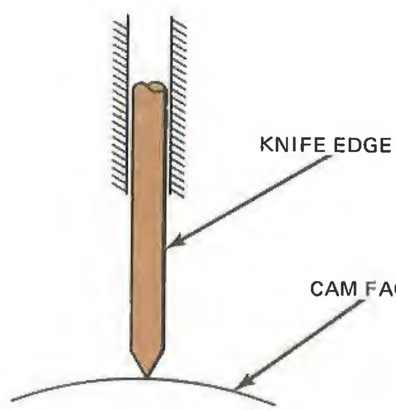


Fig. 10-11 Knife-Edge Follower

MATERIALS

1 Breadboard with legs and clamps
1 Piece of heavy plastic or cardboard, approximately 4 in. X 4 in.
1 Rotary potentiometer
1 Linear potentiometer
2 Collars
1 Eccentric cam, 2 in. OD with 1/4 bore hub 0.5 in. from center
2 Shaft hangars, 1-1/2 in.
1 DC power supply (0–40V)
1 X-Y or strip-chart recorder

PROCEDURE

1. Inspect all components and instruments to be sure they are undamaged.
2. Connect the experimental apparatus as shown in figure 10-12.

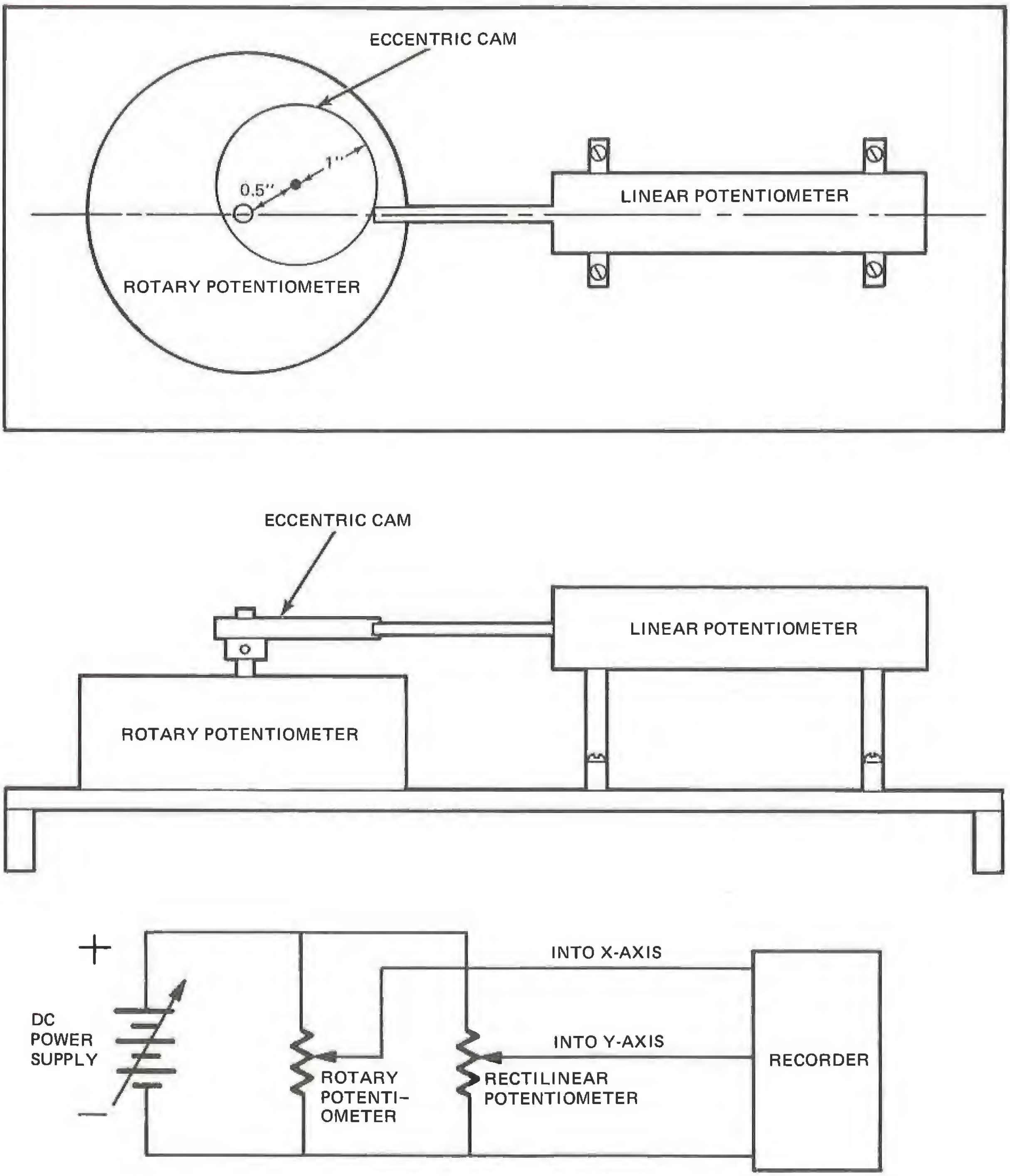


Fig. 10-12 Experimental Equipment

3. When the round wheel, called an accentric cam, is mounted as shown in figure 10-12, it will generate a sinusoidal displacement curve when the cam is turned at a constant angular velocity. Turn the equipment on and obtain the displacement curve for the eccentric cam.
4. Graphically differentiate the displacement curve to find the velocity curve.
5. Graphically differentiate the velocity curve to obtain the acceleration curve.
6. For the curve in figure 10-13, find the required cam profile, using a 2-in. diameter base circle.

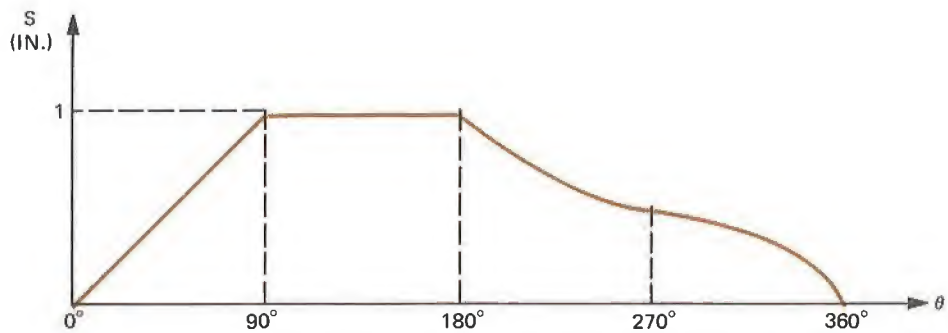


Fig. 10-13 Cam Profile For Step 6

7. Transfer the cam profile from step 6 to a piece of heavy plastic or cardboard.
8. Cut the profile of the cam out, and drill a hole in its center the same size as the potentiometer hub.
9. Using collars above and below the cam to hold it in place, install it on the potentiometer shaft, using the same setup as in figure 10-12.
10. Operate the mechanism through one complete cycle of motion to obtain the displacement curve.
11. Sketch the velocity and acceleration curves.
12. Visually compare the two displacement curves and draw them on one plot.

ANALYSIS GUIDE. In your own words, what do you think caused major errors in finding the two cam profiles? You should consider such things as human error, equipment error, and lack of perfect contact of the linear potentiometer at the cam face.

PROBLEMS

1. Would you use the cam profile you found in step 6 for high angular velocities? If not, why?

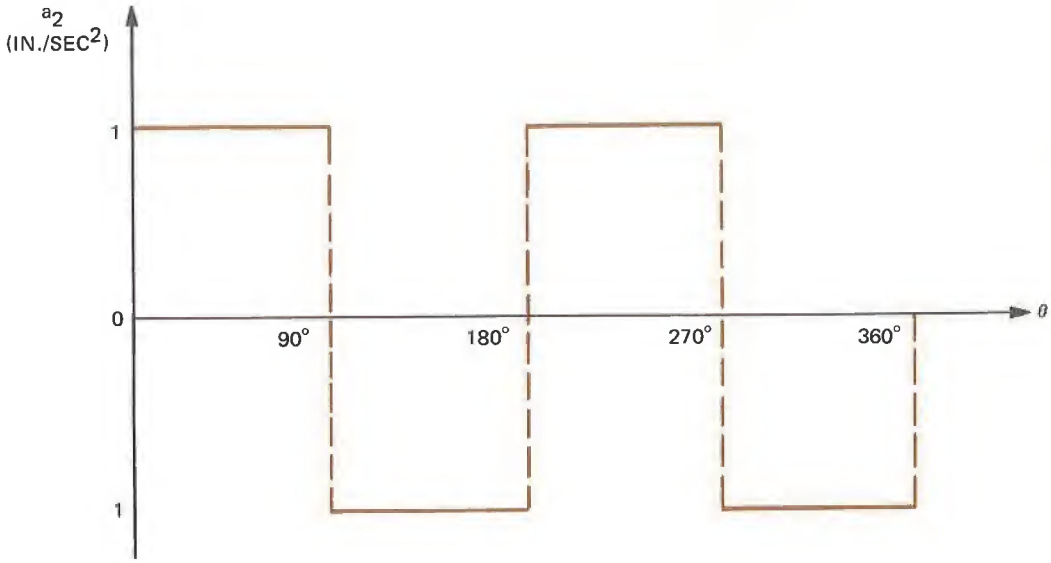


Fig. 10-14 Acceleration Curve For Problem 4

2. What single part of the curve would you change to eliminate major damage to the mechanism? Why?
3. To eliminate *all* damage from acceleration of the follower, draw the curve of figure 10-13, modified in the desired manner.
4. Find the cam profile for the acceleration curve of figure 10-14, with a 2 in. base circle. (Velocity and displacement curves may be found by inspection.)
5. Would you use the cam profile of problem 4 for low or high speed operation? Why?
6. Modify the displacement curve in figure 10-15 so that acceleration is minimized.

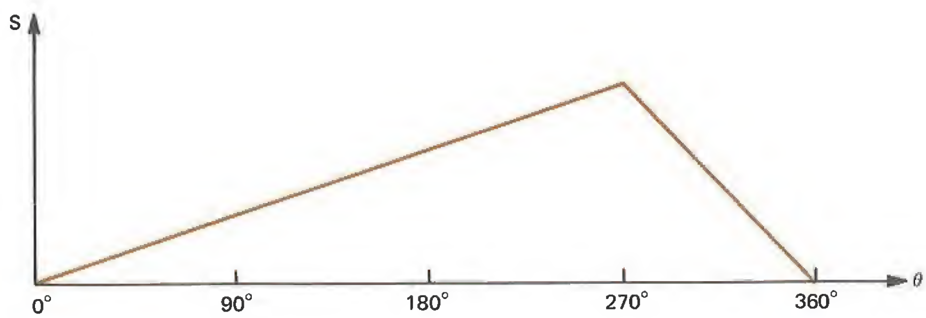


Fig. 10-15 Displacement Curve For Problem 6

experiment 11 CAM WITH DOUBLE FOLLOWER

INTRODUCTION. Double followers are commonly used with cams to amplify displacement. In this experiment we shall examine how the characteristic curve changes when the displacement is amplified.

DISCUSSION. When displacement curves are amplified, the general shape remains the same. However, the slopes of the curves change, leading to a greater difference in the derivative curves as illustrated by the *original* and *amplified* characteristic curves in figure 11-1.

Amplified curves may be obtained by several methods, some of which are shown in figure 11-2.

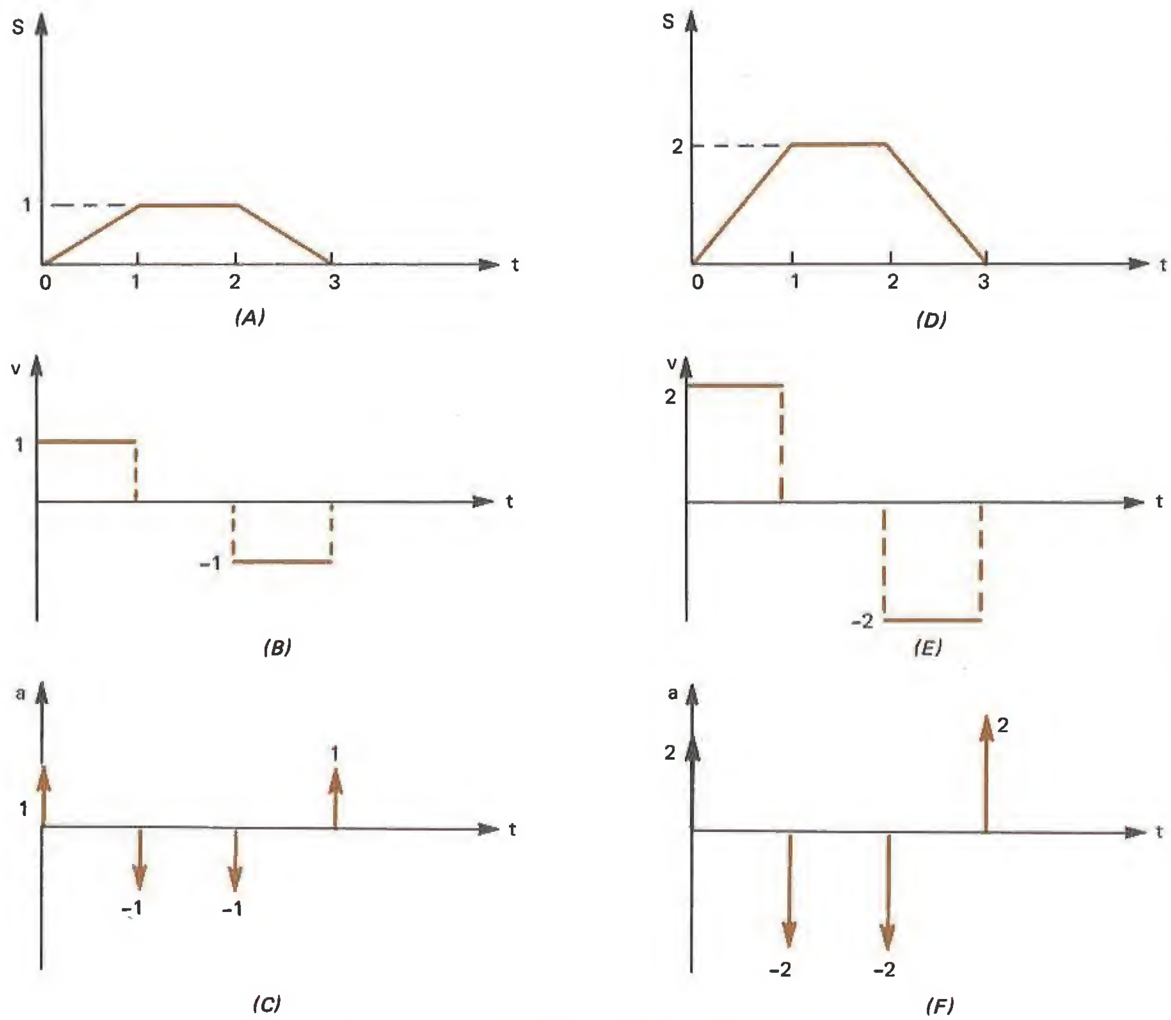


Fig. 11-1 Amplified Displacement Curve
(A), (B), (C) Original
(D), (E), (F) Amplified

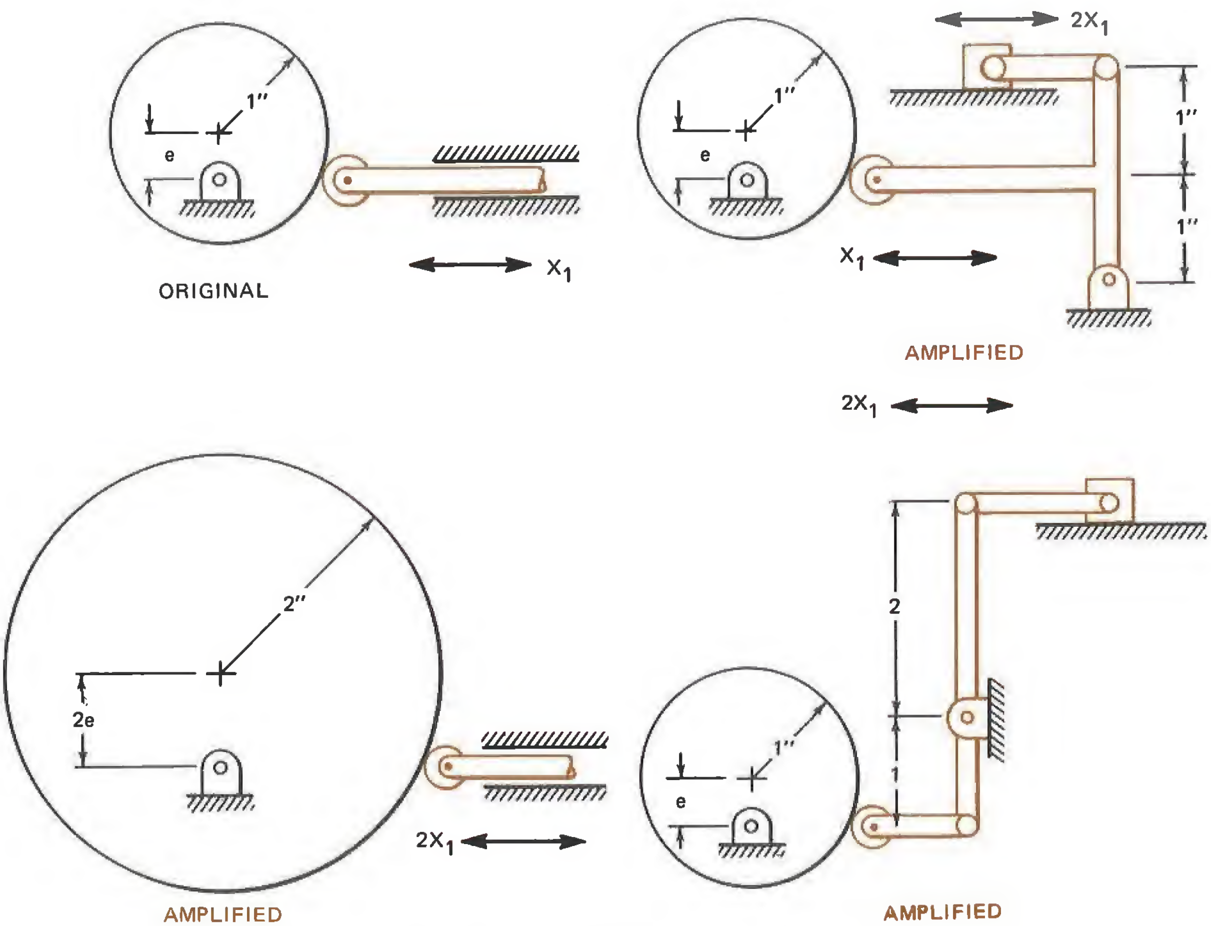


Fig. 11-2 Amplified Displacement

We observe that the two mechanisms on the righthand side of figure 11-2 will give a displacement that contains some error. However, by keeping the eccentricity, e, small, the amplitude of the sinusoidal output will also be small, and the error in displacement will then be small. If e is increased, the displacement curve is also enlarged, and the *general* shape of the curve will be the same.

Increasing e may sometimes be necessary for this mechanism if the cam size is limited and more displacement is desired. Conversely, by decreasing e, a lower displacement will result.

In dealing with double-follower cam linkages, care should always be taken that acceleration does not become too large. Since acceleration is equal to force per unit mass ($F = ma$; $a = F/m$), the time rate of change of acceleration, defined as *jerk*, is an important factor. Jerk may then be considered as being proportional to the time rate of change of force as illustrated below:

$$a = \frac{F}{m}$$

$$\text{Jerk} = \frac{da}{dt} = \frac{d(F/m)}{dt}$$

$$\text{Let } \frac{1}{m} = \text{constant} = K.$$

$$\boxed{\text{Jerk} = K\frac{dF}{dt}} \qquad \textbf{(11.1)}$$

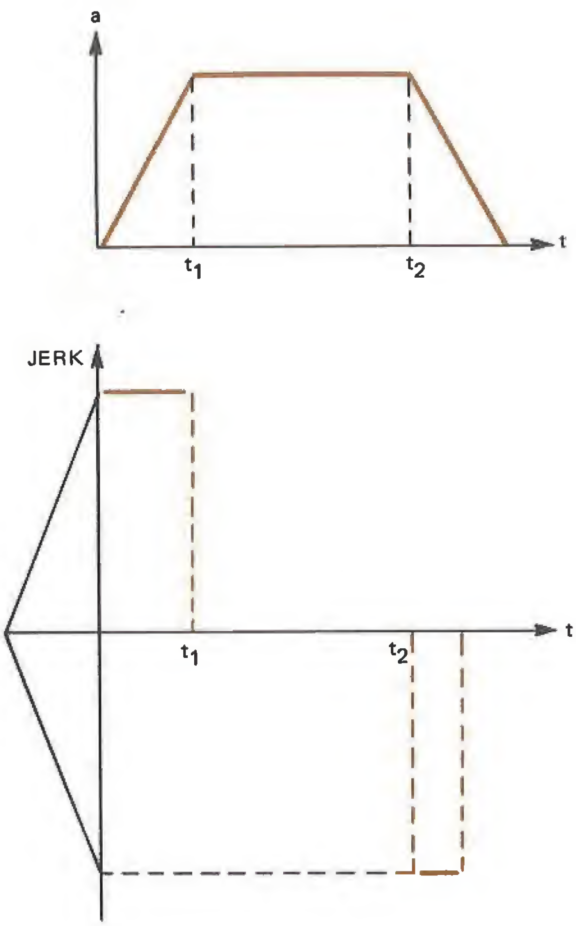


Fig. 11-3 Jerk Curvè

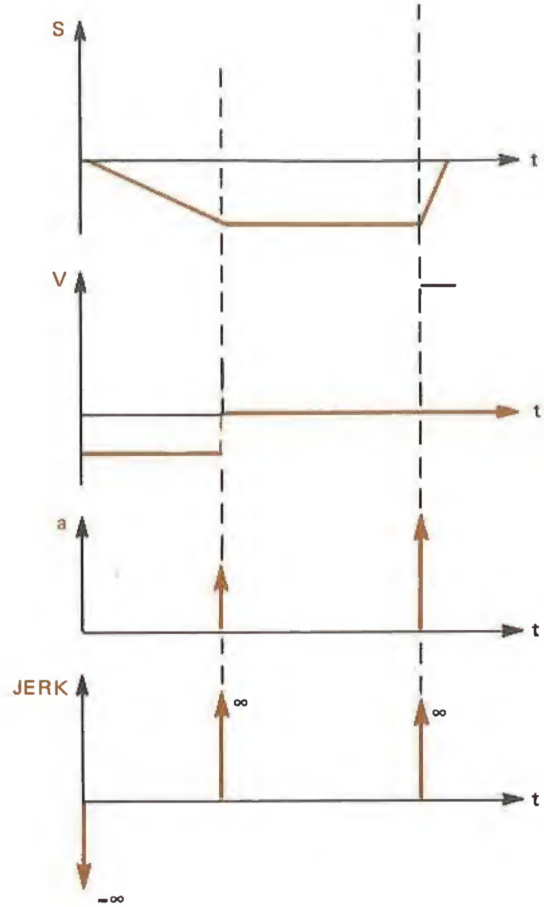


Fig. 11-4 Characteristic Curves

Thus, even though the *numerical value of acceleration* may be *low,* the *jerk* may be excessively *large*. This is illustrated by the acceleration and jerk curves in figure 11-3.

As we see from figure 11-3, at times t_1 and t_2, the jerk changes values instantaneously. In order to do this, an infinite force would be required. Since this is physically impossible, the mechanism will override, possibly causing damage to the system at high ω.

When the value of acceleration is changing instantaneously, the jerk has in infinite value and the required force is again infinite. If this mechanism is operated at even *low* values of angular velocity, it may lock briefly and will probably damage the system. The characteristic curves for this situation are illustrated by the set of curves shown in figure 11-4.,

A situation such as that in figure 11-4 should be avoided, if at all possible, if the mechanism is operated continuously. Even at low values of ω, the mechanism will experience high wear rates and have a bad vibration problem. The best way to prevent an instantaneous change in the value of jerk

is to use a continuous fourth-order displacement equation. This will result in third-order velocity, second-order acceleration, and first-order jerk curves. In this case, since the jerk equation is first-order, there can be no instantaneous change in the value of jerk. Thus, the force requirement is not infinite at any time.

If the displacement curve is third-order, the jerk curve is of order zero. This case is frequently encountered in practice since third-order curves are reasonably simple to derive, and the value of jerk may be considered to have a finite value, the slope of the acceleration curve, even though the slope of the jerk curve is zero or infinitive.

The problem of jerk is greater when the displacement curve is amplified since the slope of the acceleration curve becomes greater giving a larger value of jerk. Thus, when working with double-follower cams which enlarge the displacement curve, the value of jerk should always be checked to be sure that it does not exceed design limits.

MATERIALS

1 Breadboard with legs and clamps
1 Eccentric cam, 2-1/2 in. OD with 1/4 in. bore hub 0.5 in. from center
1 T-link, 4 in. to 2 in. ratio arms, 1/4 in. bore hub
3 Shaft hangars, one with bearings
1 Shaft, 4 in. X 1/4
1 DC power supply (0–40V)
1 Rotary potentiometer
1 Linear potentiometer
1 X-Y or strip-chart recorder
1 1 in. link

PROCEDURE

1. Check all instruments and components to be sure they are undamaged.
2. Connect the experimental equipment as shown in figure 11-5.
3. The equipment in figure 11-5 will give a sinusoidal displacement curve with a maximum amplitude of twice the eccentricity of the cam. Operate the mechanism for one cycle to obtain the displacement curve.
4. Graphically differentiate the curve from step 3 and sketch the velocity, acceleration and jerk curves.
5. Change the eccentricity from 0.5 in. to 1 in. in the experimental mechanism.
6. Operate the mechanism for one cycle to obtain the displacement curve.
7. Differentiate the displacement curve and sketch the velocity, acceleration and jerk curves.

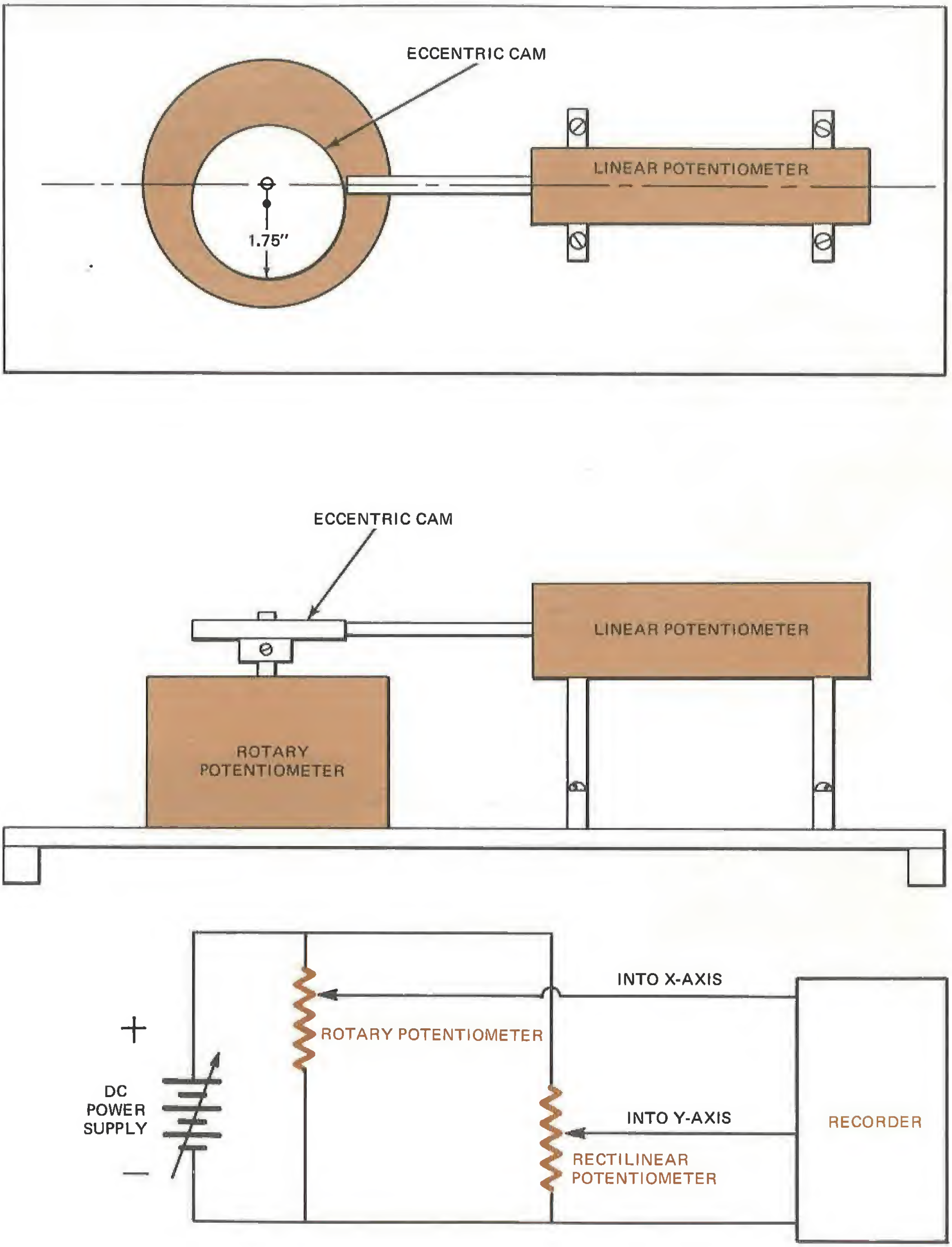


Fig. 11-5 Experimental Equipment

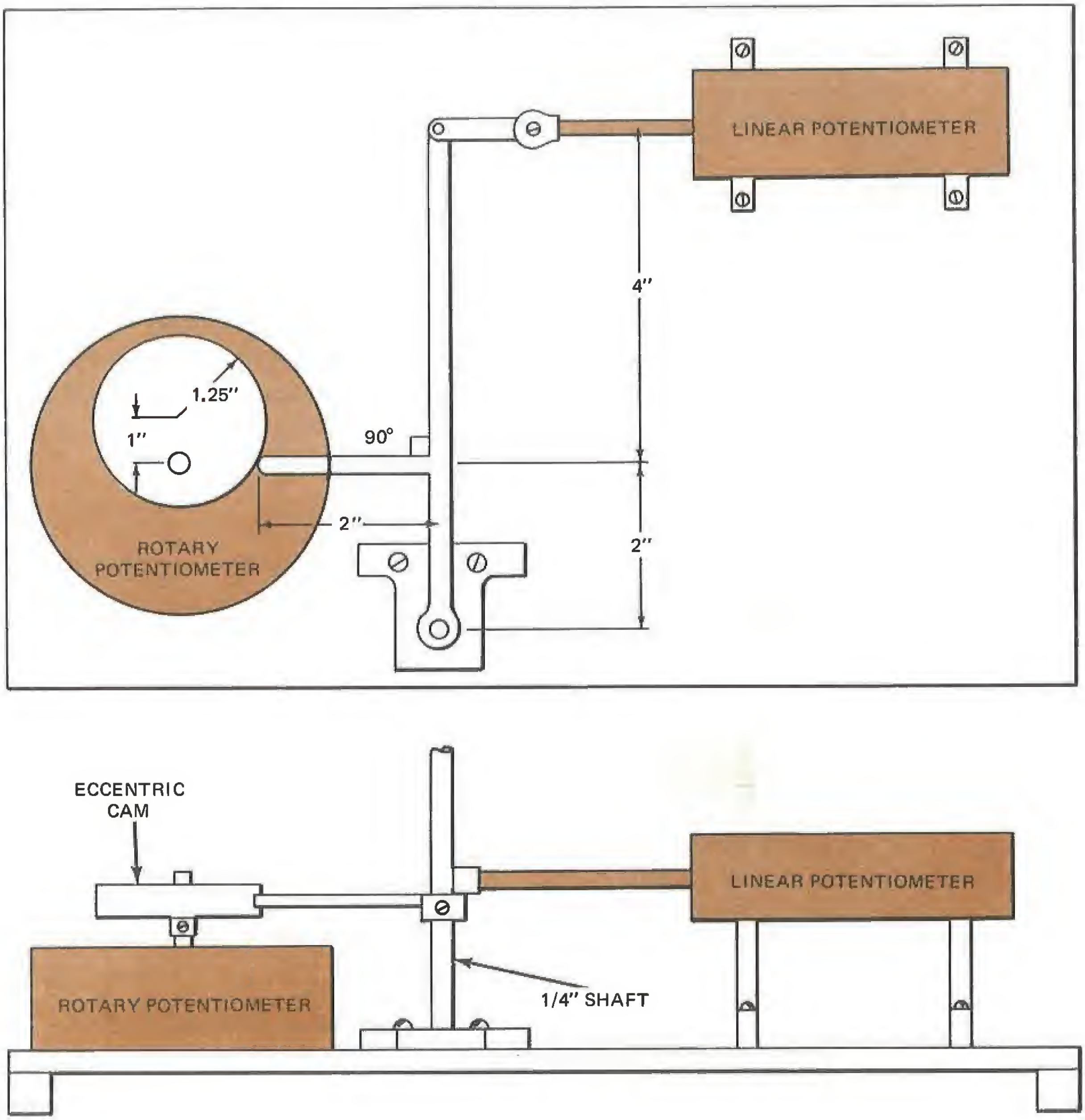


Fig. 11-6 Experimental Mechanism

8. Change the cam into a double-follower cam by connecting the linkages as illustrated in figure 11-6.
9. With the mechanism in figure 11-6, the resulting displacement curve will be an approximate amplification of that from step 6. Operate the mechanism through one cycle of motion to obtain the displacement curve.
10. Using this displacement curve, sketch the velocity, acceleration and jerk curves.

ANALYSIS GUIDE. Explain in your own words why the displacement curves in step 6 and 10 vary. Also explain why the curve in step 10 is not an *exact* multiple of the curve from step 6. Recall the section in the discussion concerning this, and also consider such factors as human error, experimental error, and equipment error.

PROBLEMS

1. Which of the three experimental arrangements would you recommend for high speed use? Why?
2. Which of the three experimental arrangements would you recommend for *only* low speed use? Why?
3. Sketch the displacement curves for points P_1 and P_2 in figure 11-7.
4. Visually compare the two curves from problem three. By what factor is S_1 multiplied to obtain S_2? Is the curve for P_2 an *exact* multiple of the one for P_1?

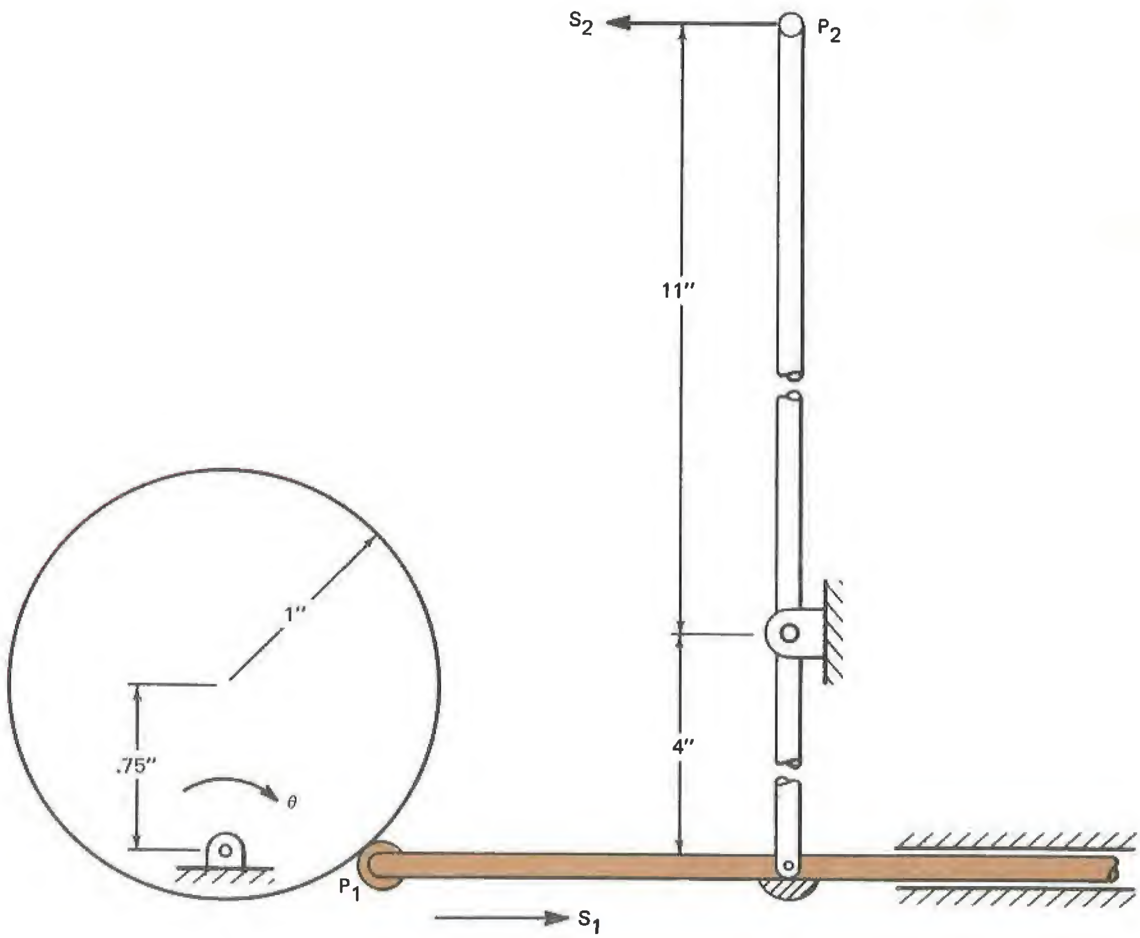


Fig. 11-7 Amplified Displacement

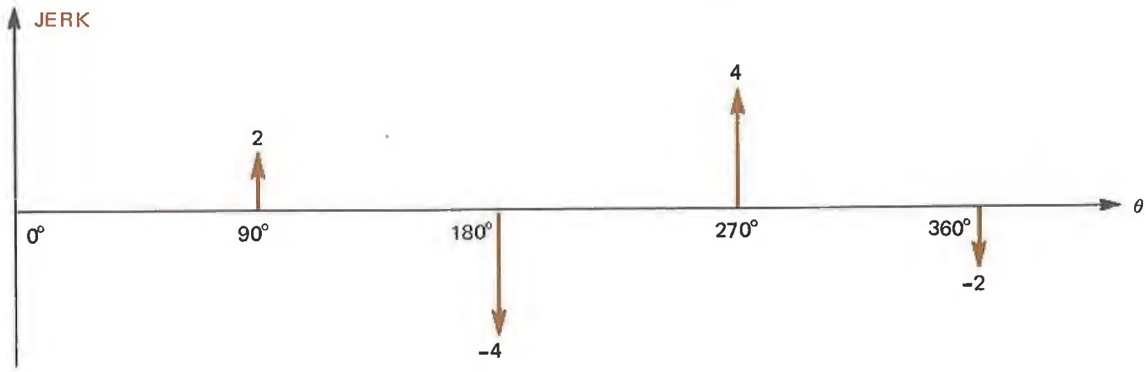


Fig. 11-8 Jerk Curve For Problem 5

5. For a mechanism similar to that in figure 11-7, sketch a displacement curve for the jerk curve in figure 11-8.
6. Find the *disk* cam profile for your displacement curve in problem 5.
7. Would you recommend this cam for high, medium, or low speed operation? Why?

experiment 12 COMPLEX MOTION CAMS

INTRODUCTION. Complex motions can be performed by a fairly simple arrangement of cams. As an illustration of this, we will investigate how two cams can be used to generate two-dimensional motion.

DISCUSSION. Although a cam can generate a wide variety of displacement curves, it is essentially a *one-dimensional mechanism.* A follower contained in a guide slot is the closest to exact one-dimensional motion. If the displacement is very large, link-type followers will have *some* displacement on the *second* axis also, giving a motion which may be considered to be two-dimensional. However, since the link has to pivot about the fixed end, the motion performed is an arc of a circle, and movement on the second axis is just the deviation of the circular arc from a straight line. For small displacements or large link lenghts, this variation is very small.

A single cam can be used as the basic source to provide movement in two directions by adding another linkage. Although the cam itself does not *directly* provide the two-dimensional movement in this case, this is at least a *means* of obtaining two-dimensional motion indirectly from a single cam. This cam and linkage is shown in figure 12-1.

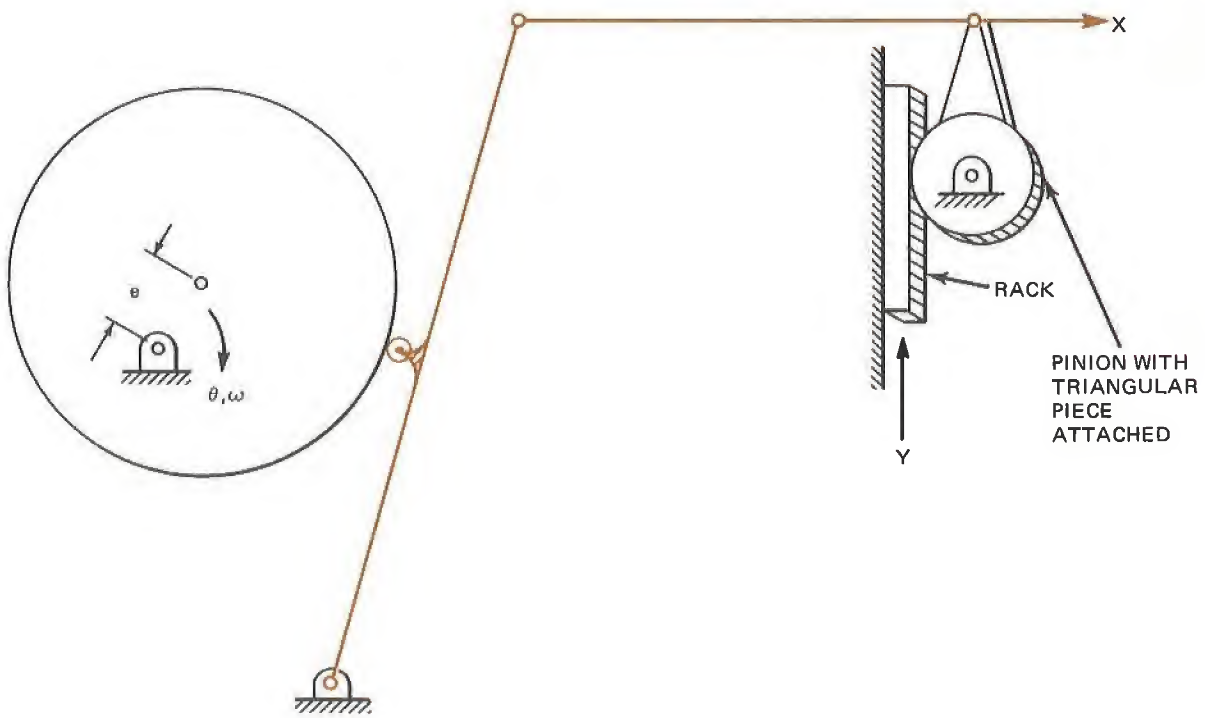


Fig. 12-1 Two-Dimensional Motion Mechanism

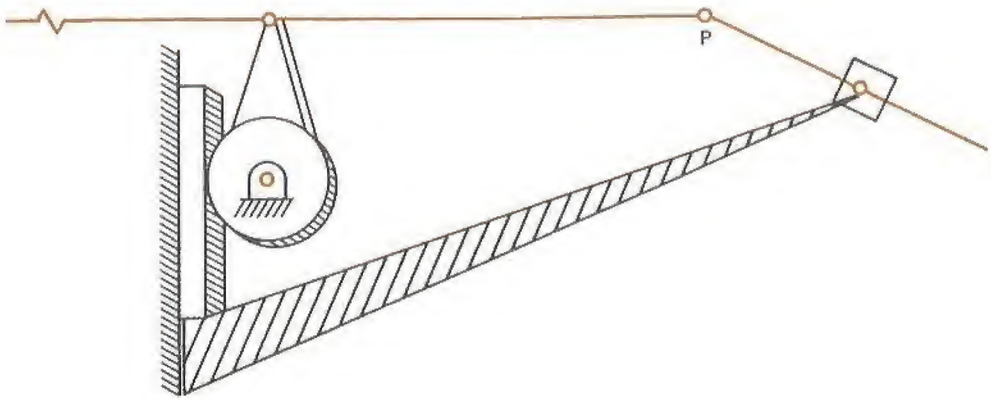


Fig. 12-2 Two-Dimensional Motion

In figure 12-1, we observe that the X and Y components are a function of each other, and each is a function of θ. However, no *single* point in the figure has equal or near equal components of displacement along *both* the X and Y axis. To achieve this, we need to add another linkage as illustrated by the partial diagram of figure 12-2.

We see from figure 12-2 that, with the slider linkage added to the mechanism of figure 12-1, point P will have two-dimensional motion. However, designing this linkage to obtian a *particular* motion of point P would be rather difficult.

A mechanism which will also give two-dimensional motion is shown in figure 12-3.

This linkage is slightly more complex, and again, the linkage is difficult to find for a particular motion curve of point P.

In general, linkages which give two-dimensional curves of motion when connected

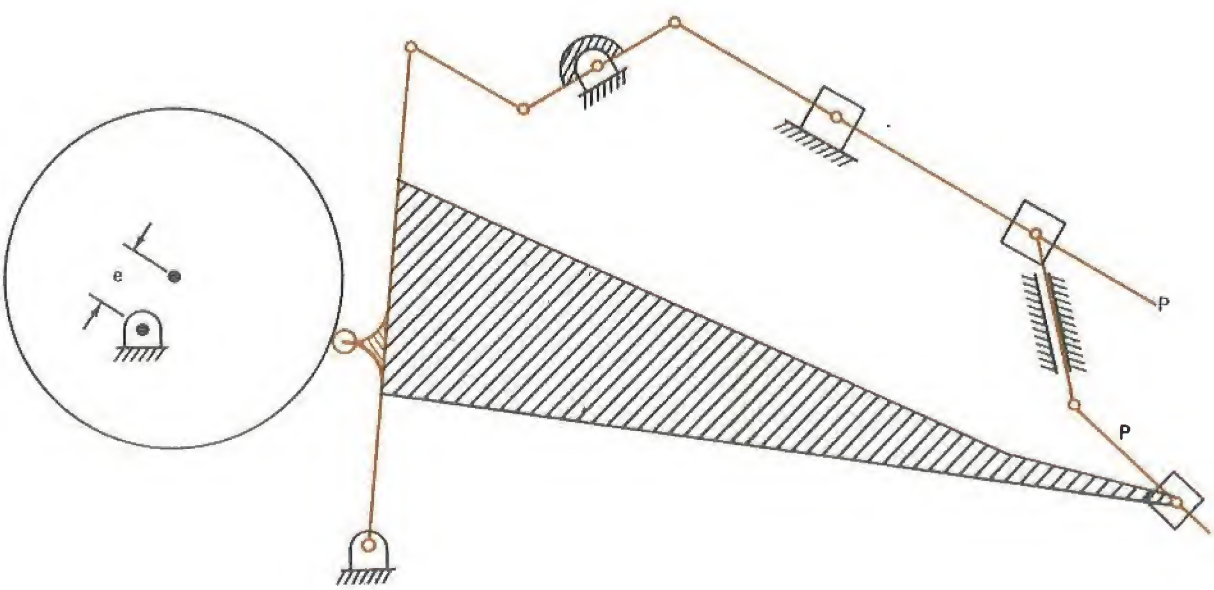


Fig. 12-3 Two-Dimensional Motion

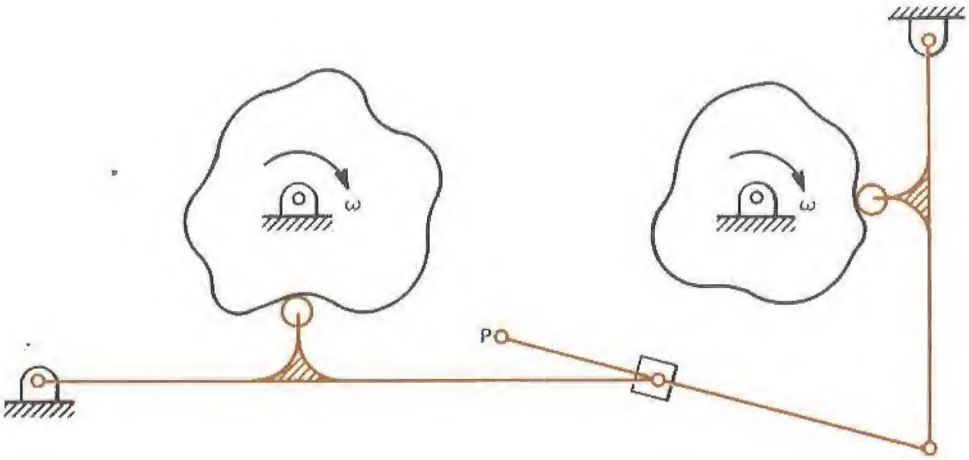


Fig. 12-4 Two-Dimensional Motion

to a cam are difficult to derive or predict for a *particular* displacement curve of a point.

One way to avoid this difficulty is to use *two* cams with the contact point paths perpendicular to each other. This is shown by the linkage in figure 12-4, where point P will have *both* X and Y components of displacement.

By comparing figure 12-4 with figures 12-2 and 12-3, we observe that the linkage of figure 12-4 is much simpler and would be easier to synthesize for a particular displacement curve for point P.

However, the linkage still leaves much to be desired in simplicity as a comparison of figure 12-4 with figure 12-5 will show.

The two cam profiles and two link lengths are the four important variables in figure 12-5. By *assuming* some constant *length* for the links, the unknown variables

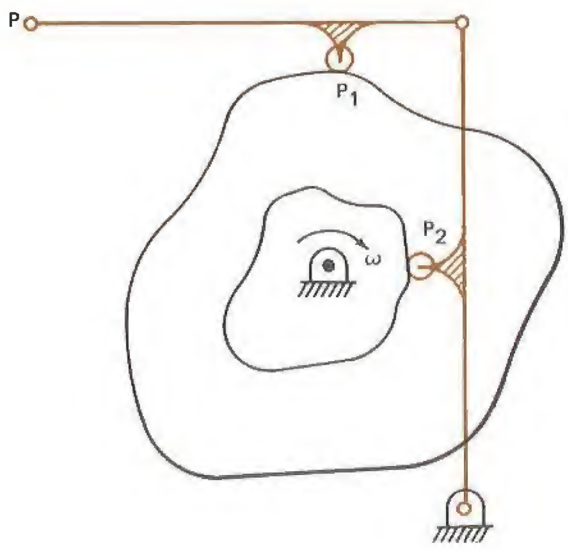


Fig. 12-5 Two-Dimensional Motion

are reduced to only the two cam profiles, and these can then be found for a displacement curve.

One procedure for finding the cam profiles is to use the links of known length and a piece of paper in the place of the two cams. The desired displacement curve is drawn on another piece of paper and point P moved along the curve at incremental displacements. The second piece of paper is rotated at corresponding angular increments. Points P_1 and P_2 are marked for each corresponding increment of motion of point P. This procedure is illustrated in figure 12-6 for a simple curve.

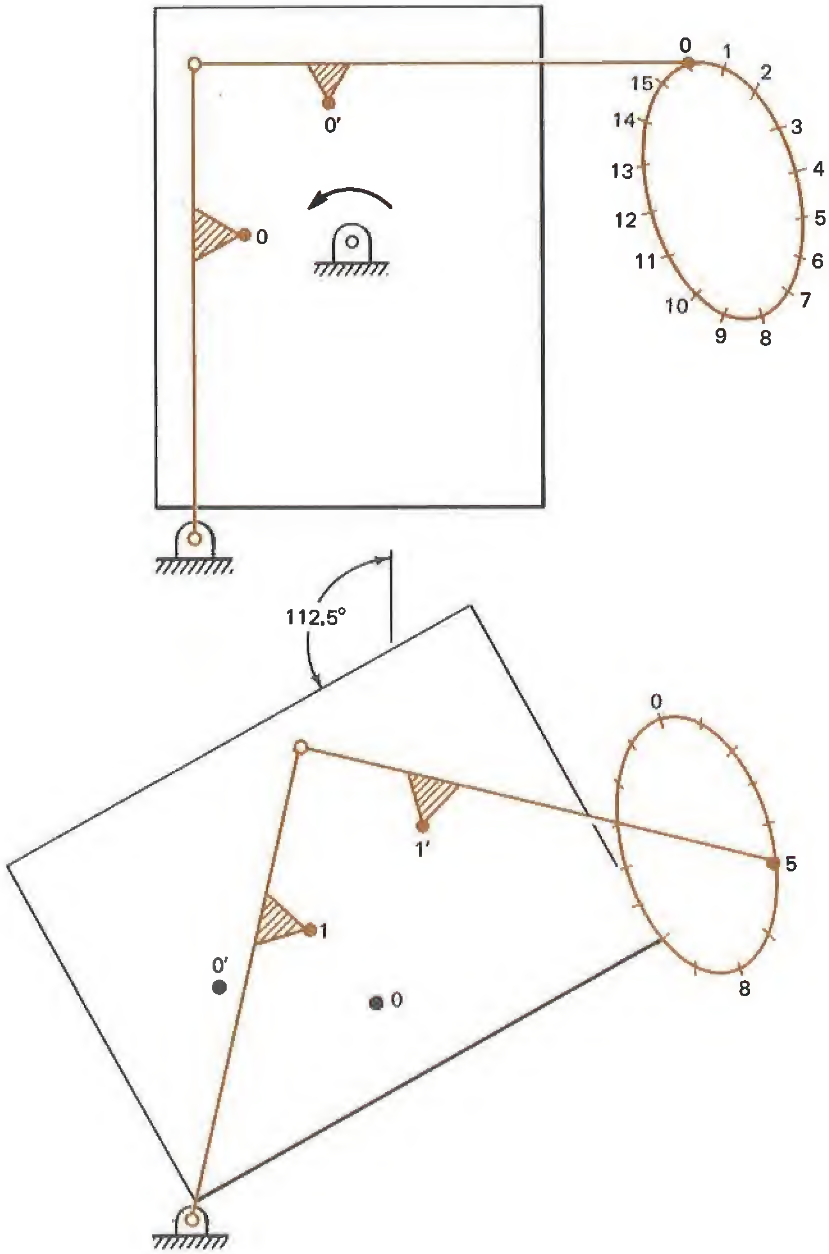
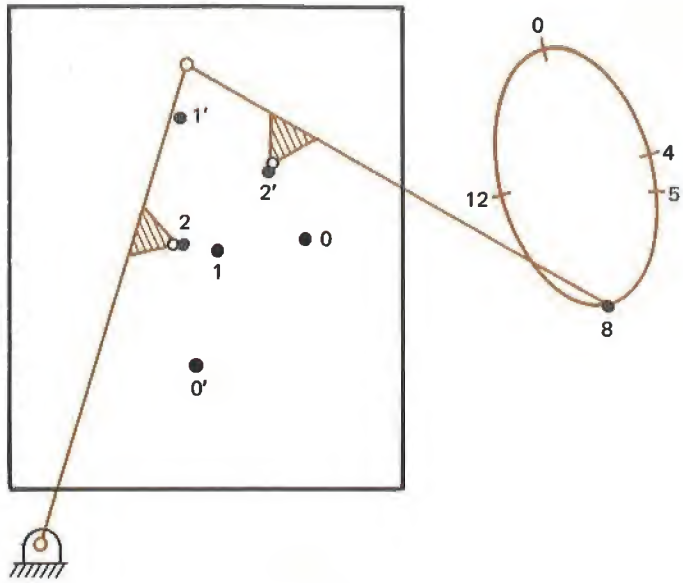


Fig. 12-6 Two-Dimensional Motion Cam Profile



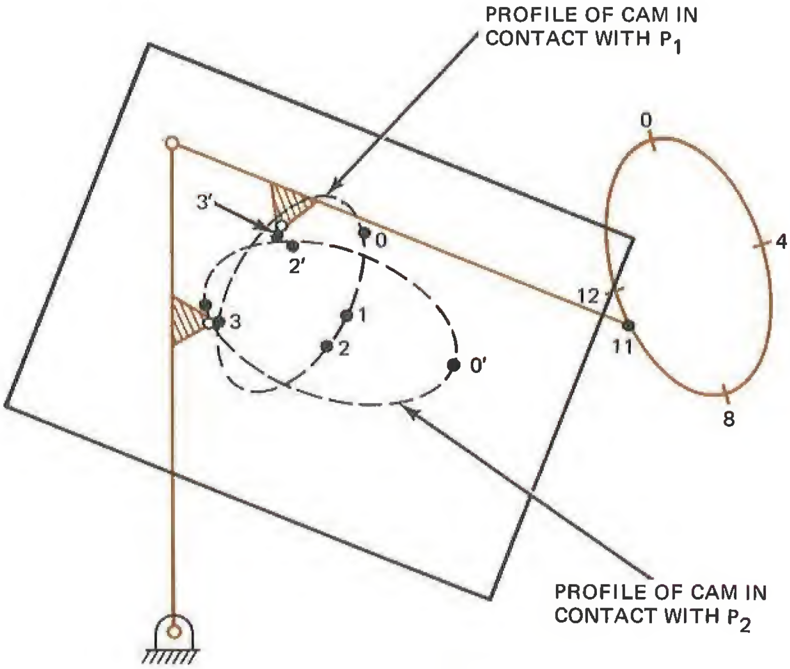


Fig. 12-6 Two-Dimensional Motion Cam Profile (Cont'd)

After a number of points such as 1 and 2 are marked, they are connected by a smooth curve. The two resulting curves are the two cam profiles, and these can be transferred to the cam material, and the cam machined.

This linkage is the simplest way to get two-dimensional movement from cams. The same limitations apply to the double-cam arrangement as to single cams. Care must be taken that the links do not float, and jerk (da/dt) should be kept to a minimum.

MATERIALS

1 Breadboard with clamps
1 Eccentric cam, d = 2 in., e = 0.5 in.
1 3 in. connecting link
1 Rotary potentiometer
1 Rectilinear potentiometer
1 DC power supply
1 Cardboard square, 4 in. X 4 in.
1 Engineer's scale
1 Straightedge
1 Drafting pencil
1 Compass
1 Protractor
1 Universal hub, capable of holding 2 wheels
1 Eccentric cam, d = 2-1/2 in., e = 0.5 in.
1 Cam follower (1.5 in.)
1 Cam follower (3.5 in.)
1 Strip chart or X-Y recorder

PROCEDURE

1. Check all experimental components to be sure they are undamaged.
2. Cut the two follower links out and connect the linkage as shown in figure 12-7, with the eccentricity for both cams = 0.5 in.. Use an angle of 20° between the mounting point and centers of the two cams.

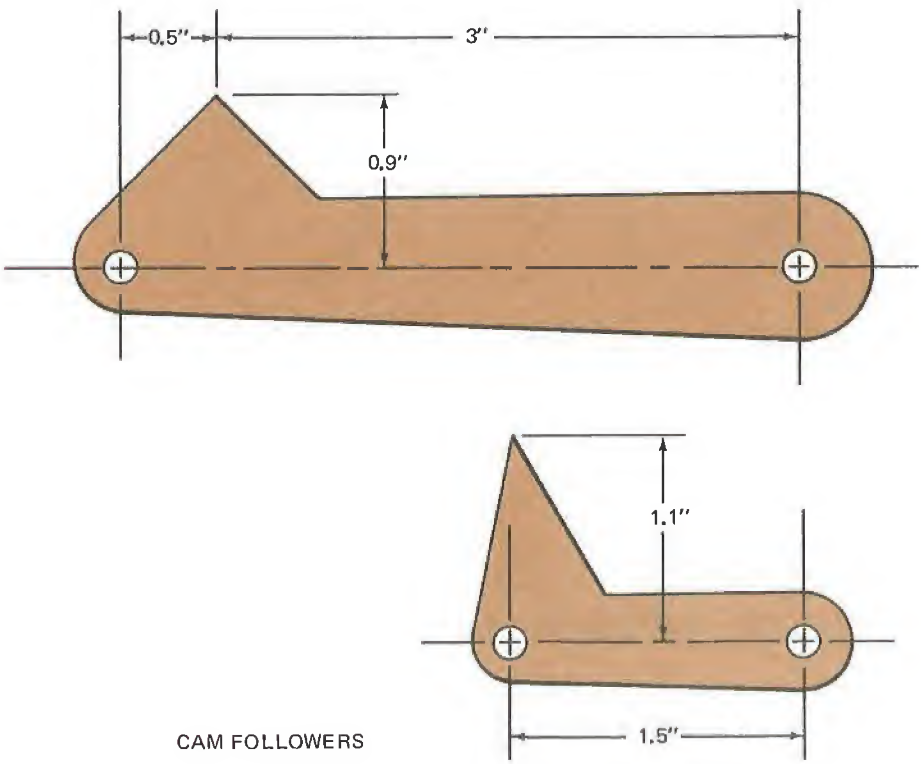
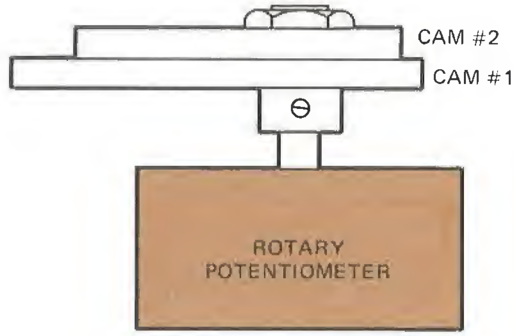


Fig. 12-7 Experimental Mechanism



ROTARY POTENTIOMETER AND CAM ASSEMBLY

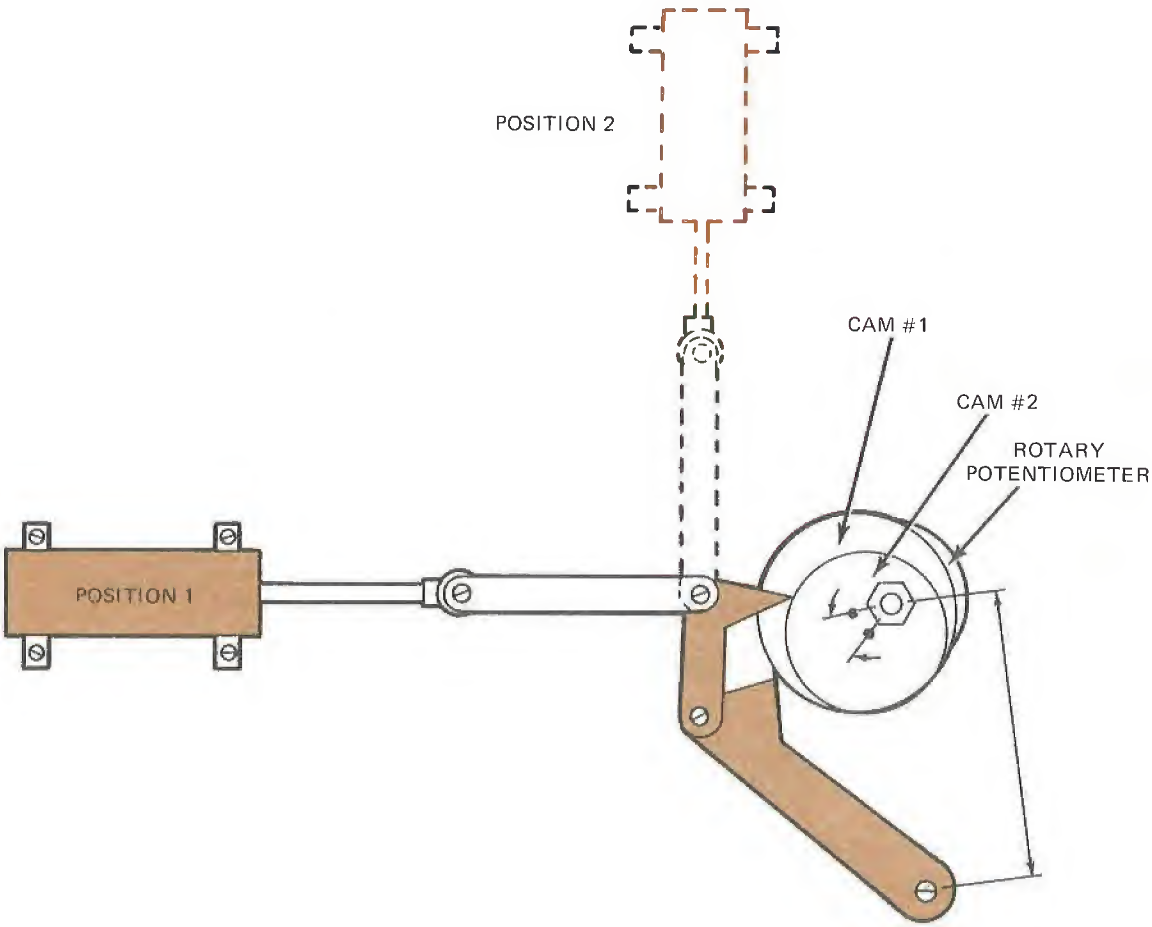


Fig. 12-7 Experimental Mechanism (Cont'd)

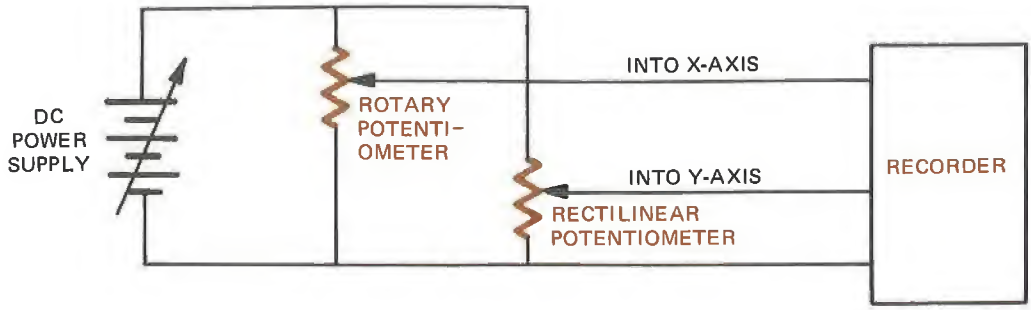


Fig. 12-7 Experimental Mechanism (Cont'd)

3. Operate the mechanism for one cycle and obtain the displacement curve for point P by marking the movement on paper as the cams are turned.
4. Turn the equipment on and operate the cams for one cycle with the linear potentiometer in position 1 to obtain the X-axis displacement plot.
5. Reconnect the linear potentiometer in position 2 and obtain the Y-axis displacement curve. Again, zero the linear potentiometer in the mid-point of the deflection.
6. Plot the X and Y displacements for each θ to find the path of motion.
7. compare the curves from steps 3 and 6.
8. Change the angle between the mounting point and the centers of the cam to 340°.
9. Repeat steps 3 through 7.
10. Compare the two curves from step 6 and the similar one from step 9.

ANALYSIS GUIDE. Discuss possible reasons why the curves in steps 3 and 6 are *not* exact duplicates. You should consider such things as human error in measurement, and induced error in the way X and Y components of displacement were found, such as the effect of a sign change in plotting the X or Y components.

PROBLEMS

1. For what operational speed would you recommend the first linkage?
2. Plot the path of motion for point P in figure 12-8. Assume both cams are turned at the same rate.
3. Repeat problem 2, with the 0.75-in. radius cam turning *twice as fast* as the 1-in. radius cam.
4. By inspection of the path of motion in figure 12-9 for an arrangement similar to that in figure 12-8, what is the eccentricity for each of the cams? (Assume the follower links are infinitely long.)
5. Will there be any difference in the displacement curves for the two eccentric cams in figure 12-10? Why?

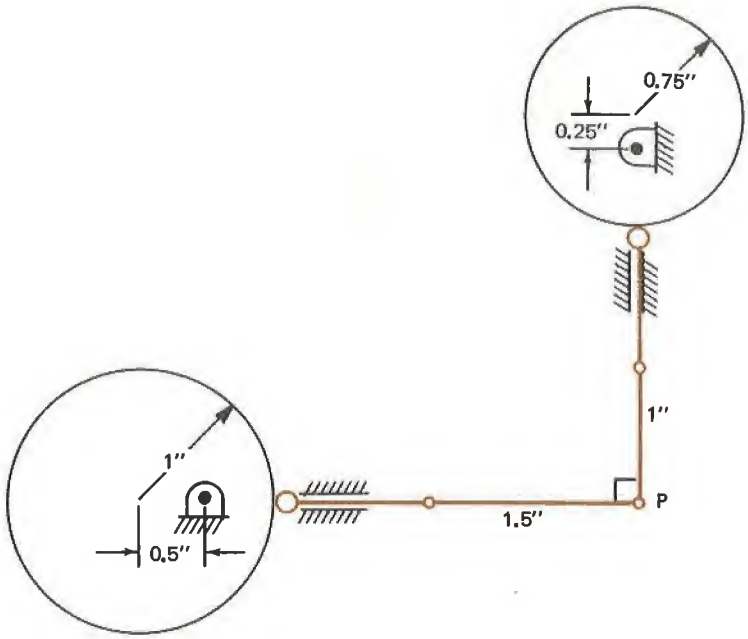


Fig. 12-8 Double Cams

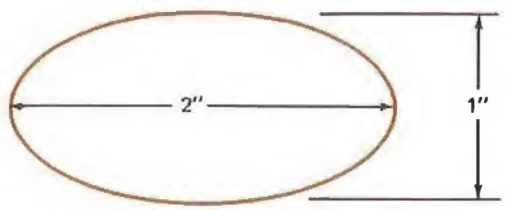


Fig. 12-9 Path of Motion

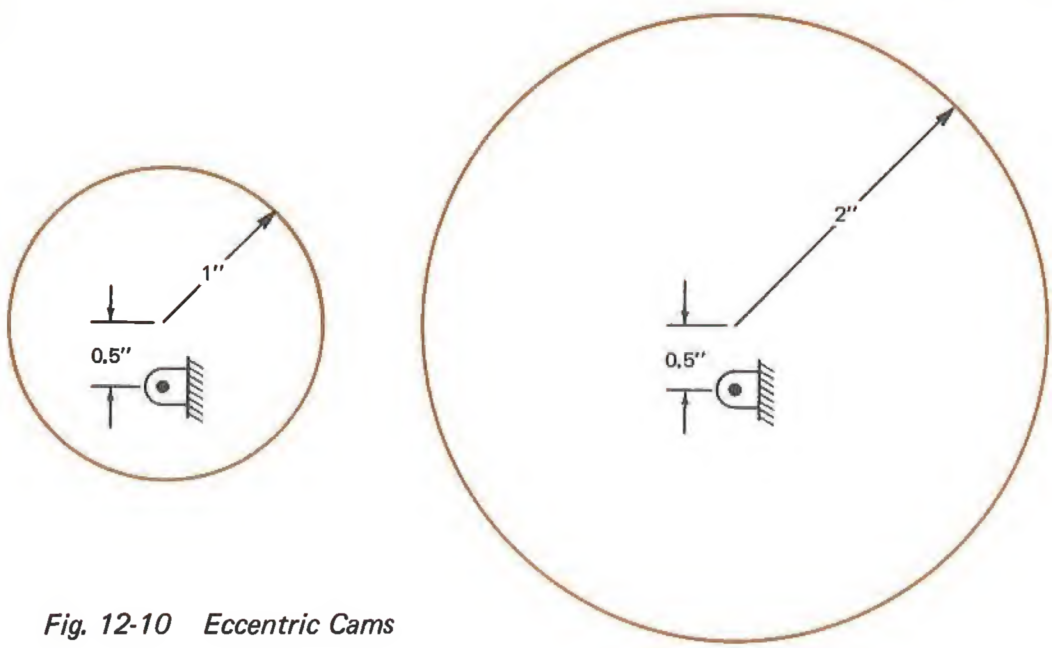


Fig. 12-10 Eccentric Cams

experiment 13 VELOCITY MULTIPLICATION WITH GEARS

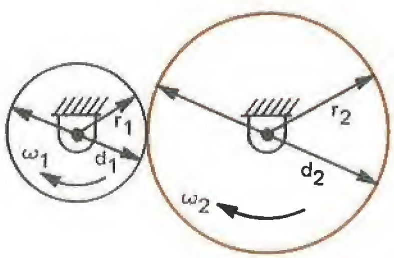
INTRODUCTION. One of the uses of gears is that of velocity multiplication. This is accomplished at the expense of the torque transmitted. In this experiment we shall examine how the characteristic curves change when a linkage is "geared up".

DISCUSSION. Gear trains are often used to transmit *rotary* motion from one point to another when distance between the points is not too great. The angular velocity transmitted to the second point often has a different value than the driving velocity and is sometimes capable of being varied. Examples of this are the differential and transmission of a car. The differential transmits the angular velocity of the driveshaft to the axles, and the velocity ratio of input velocity to output velocity remains a constant.

The transmission, however, is capable of varying the velocity ratio by shifting gears and changing the diameter of the gears used to transmit the motion. Another example is the long shaft in old factories or machine shops which is driven at a constant velocity. From this shaft belts are connected with various machines or tools. The machines may have a gear train to change the angular velocity even more than allowed by the belt-driven wheels. They may also have a transmission to *vary* the velocity ratio if it is desired to do so.

We recall that the velocity ratio is equal to the gear teeth number, radius, or diameter ratio of the input shaft to the output shaft, as illustrated in figure 13-1.

The velocity ratio remains the same if we insert an idler gear between the two gears of figure 13-1 as shown by the development in equation 13.2 for figure 13-2.

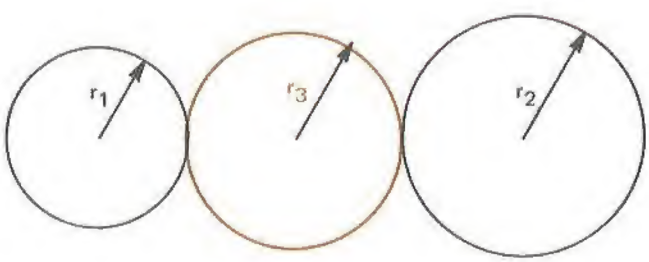


VELOCITY RATIO
$= \omega_2/\omega_1$
= GEAR RATIO
$= r_1/r_2$
$= d_1/d_2$
$= t_1/t_2$

t = NUMBER OF GEAR TEETH

(13.1)

Fig. 13-1 Velocity Ratio



$$\boxed{\left|\frac{\omega_2}{\omega_1}\right| = \left|\left(\frac{r_1}{r_3}\right)\left(\frac{r_3}{r_2}\right)\right| = \left|\frac{r_1}{r_2}\right|} \tag{13.2}$$

Fig. 13-2 Idler Gear

Thus, the velocity ratio is again seen to be the ratio of input to output gear radius or diameter ratio, and the intermediate idler gear does not affect the velocity ratio at all.

When *two* or more intermediate gears are used with two gears on one shaft, the velocity ratio changes and becomes the drive gears' radii product divided by the driven gears' radii product. This is shown in figure 13-3 and equation 13.3.

In equation 13.3 the number of teeth, t, of each gear may also be substituted in place of the radius or diameter.

Magnitude signs have been placed in equations 13.2 and 13.3 because we have not specified a sign convention. Normally, we let gear train velocity ratios be *positive* if the input and output gears rotate in the *same* direction. A *negative* sign is assigned if they have *opposing* directions of rotation. This convention is illustrated in figure 13-4.

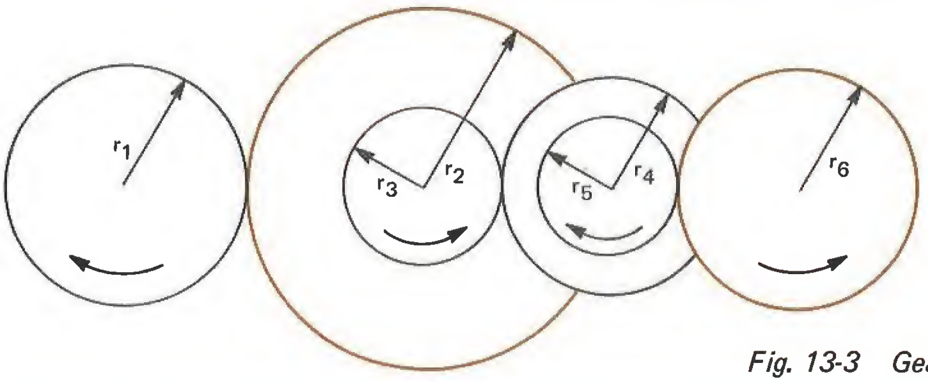
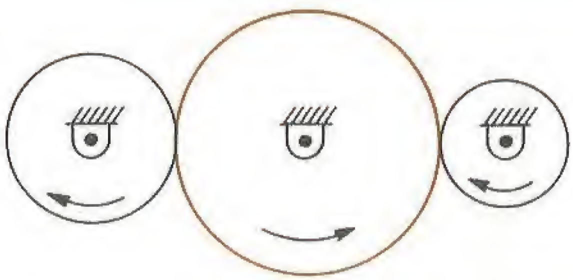


Fig. 13-3 Gear Train

$$\left|\frac{\omega_6}{\omega_1}\right| = \left|\left(\frac{r_1}{r_2}\right)\left(\frac{r_3}{r_4}\right)\left(\frac{r_5}{r_6}\right)\right| = \left|\left(\frac{d_1}{d_2}\right)\left(\frac{d_3}{d_4}\right)\left(\frac{d_5}{d_6}\right)\right| = \left|\left(\frac{\omega_2}{\omega_1}\right)\left(\frac{\omega_4}{\omega_3}\right)\left(\frac{\omega_6}{\omega_5}\right)\right| = \left|\frac{\omega_6}{\omega_1}\right| \qquad (13.3)$$

POSITIVE VELOCITY RATIO



NEGATIVE VELOCITY RATIO

Fig. 13-4 Sign Convention for Gear Trains

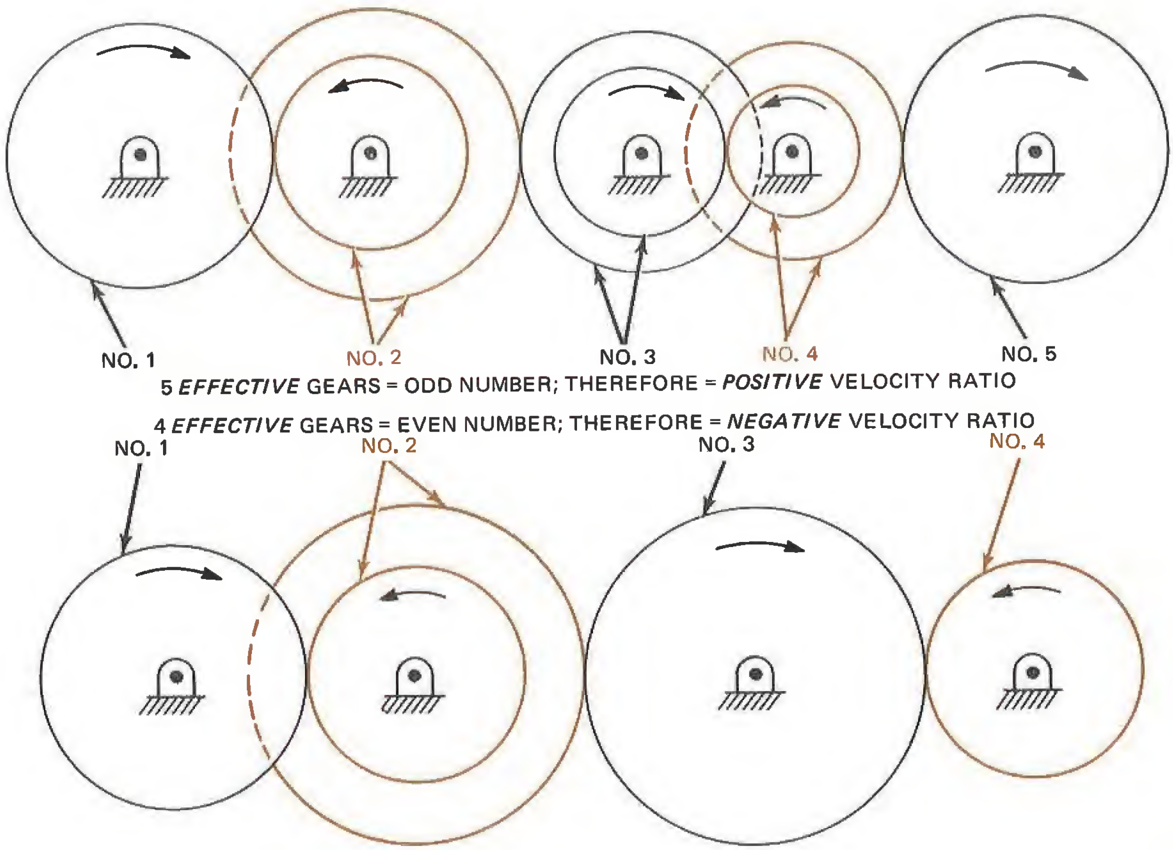


Fig. 13-5 Effective Number of Gears

We can formulate, by inspection of figure 13-4, a *rule of signs* for simple gear trains. By counting the two gears on a common shaft, or two chain-driven gears as one gear, we see that a system containing an *odd* number of gears has a *positive* velocity ratio, and an *even* number of gears will result in a *negative* number of gears. This procedure is illustrated in figure 13-5.

Applying this convention to equation 13.3 for figure 13-3, the result is

$$\boxed{\begin{aligned}\frac{\omega_6}{\omega_1} &= -\frac{r_1 r_3 r_5}{r_2 r_4 r_6} = -\frac{d_1 d_3 d_5}{d_2 d_4 d_6} \\ &= -\frac{t_1 t_3 t_5}{t_2 t_4 t_6} = -\frac{\omega_2 \omega_4 \omega_6}{\omega_1 \omega_3 \omega_5}\end{aligned}} \qquad (13.4)$$

In designing a simple gear train then, if we know the directions of rotation of the input and output gears, we know whether there should be an *even* or *odd* effective number of gears. If the input and output gears' angular velocity and diameter are known, we may solve for the diameter ratios of the intermediate gears by assuming some information. Normally, the distance between the centers of the input and output shafts will also be known; therefore, we may find the distance between the gears in which the intermediate gears must fit if the centers all lie on a straight line.

To illustrate this, let us consider the case of two gears, separated at their inside edges by a distance of one foot. The gears are to turn in *opposite* directions; input d, ω = 6 in., 100 RPM; output d, ω = 12 in., 400 RPM.

Opposite directions = even number of gears.

From equation 13.4,

$$\frac{\omega_{out}}{\omega_{in}} = \frac{400}{100} = \frac{\omega_{driven}}{\omega_{drivers}} = 4$$

and

$$\frac{\omega_{out}}{\omega_{in}} = 4 = -\left(\frac{d_{in}}{d_{out}}\right)\left(\frac{d_{drivers}}{d_{driven}}\right)$$

$$= -\left(\frac{6}{12}\right)\left(\frac{d_{drivers}}{d_{driven}}\right) = -0.5\left(\frac{d_{drivers}}{d_{driven}}\right)$$

Now we see that

$$\frac{\omega_{driven}}{\omega_{driver}} = 4 = -0.5\left(\frac{d_{drivers}}{d_{driven}}\right)$$

where $(d_{drivers} + d_{driven}) = 1$ ft or, $d_{drivers} = (1 \text{ ft} - d_{driven})$

At this point we will make the following assumptions:

There will only be *two* intermediate gears, and all gear centers lie on a straight line.

Therefore, *one* solution would be:

$$0.5\left(\frac{d_{driver}}{d_{driven}}\right) = 0.5\left(\frac{1 \text{ ft} - d_{driven}}{d_{driven}}\right) = 4$$

$$1 \text{ ft} - d_{driven} = 8\ (d_{driven})$$

$$9d_{driven} = 1 \text{ ft}$$

$d_{driven} = d_3 = \frac{1}{9}$ ft
$d_{driver} = d_2 = \frac{8}{9}$ ft

To check this answer,

$$\frac{\omega_{out}}{\omega_{in}} = \frac{\omega_4}{\omega_1} = 4 = -\left(\frac{d_{in}}{d_{out}}\right)\left(\frac{d_{driver}}{d_{driven}}\right)$$

$$= 0.5\left(\frac{8/9}{1/9}\right) = 4$$

If we had desired to put three or more intermediate gears in the gear train of figure 13-6, we would have needed either to have more information or to make some assumptions. Normal design practice is to use

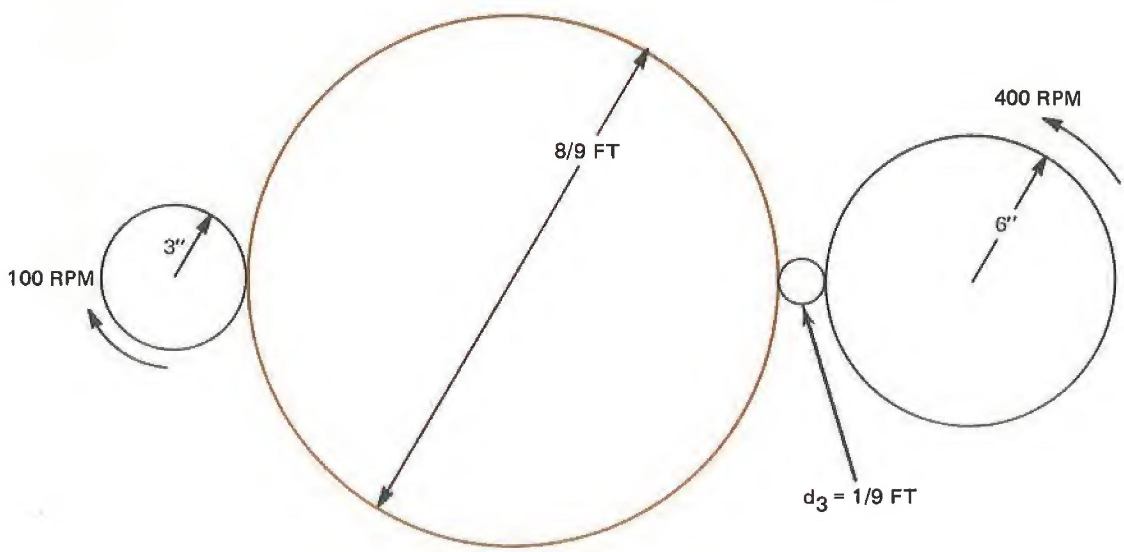


Fig. 13-6 Example Problem Solution Sketch

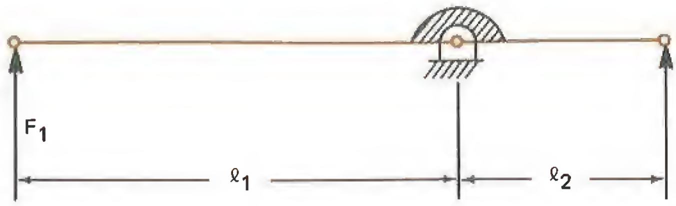


Fig. 13-7 Mechanical Advantage in Linkages

compromises between the gear size, placement, strength, number of gears, etc. Possibly the most important element in designing a gear train is to use gears that are *commercially available* when possible.

The concept of *mechanical advantage* is often important. We recall that for linkages there is an inverse relationship between the two applied forces and the distances to the pivot point as illustrated by figure 13-7 and equation 13.5.

$$\frac{F_1}{F_2} = \frac{\ell_2}{\ell_1} = \text{Mechanical Adv.} \qquad (13.5)$$

if F_2 is the input.

We may now extend this to torque amplification in gear systems, as illustrated by figure 13-8 and equation 13.6. Torque is defined as follows:

$$\boxed{\vec{\tau} = \vec{\ell} \times \vec{F}}$$

$$\boxed{\frac{\tau_4}{\tau_1} = \left(\frac{\omega_1}{\omega_2}\right)\left(\frac{\omega_2}{\omega_3}\right)\left(\frac{\omega_3}{\omega_4}\right) = \frac{\omega_1}{\omega_4} = \frac{r_4}{-r_1}} \qquad (13.6)$$

$$\boxed{\tau_4 = \tau_1 \frac{\omega_1}{\omega_4} = -\tau_1 \frac{r_4}{4_1}}$$

We observe from equation 13.6 that if r_4 or ω_1 is increased, τ_4 is increased. If r_1 or ω_4 is decreased, τ_4 will decrease.

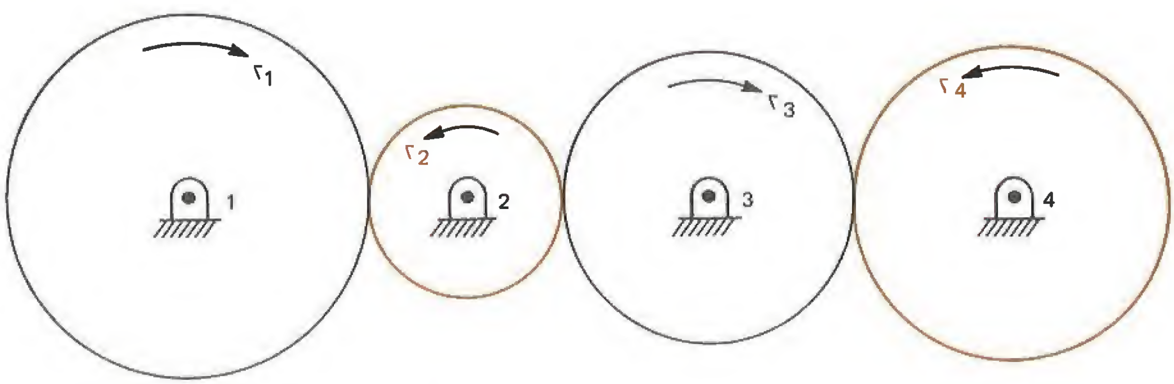


Fig. 13-8 Gear Train Torque Amplification

MATERIALS

1 Breadboard with legs and clamps
2 Bearing holders with bearings
2 Rotary potentiometers
1 0.5 in. radius gear
1 1 in. radius gear
1 1.5 in. radius gear
1 2 in. radius gear
1 DC power supply
1 X-Y or strip-chart recorder
2 Shafts 4 X 1/4

PROCEDURE

1. Check all components and instruments to be sure they are undamaged.

2. Connect the experimental apparatus as shown in figure 13-9.

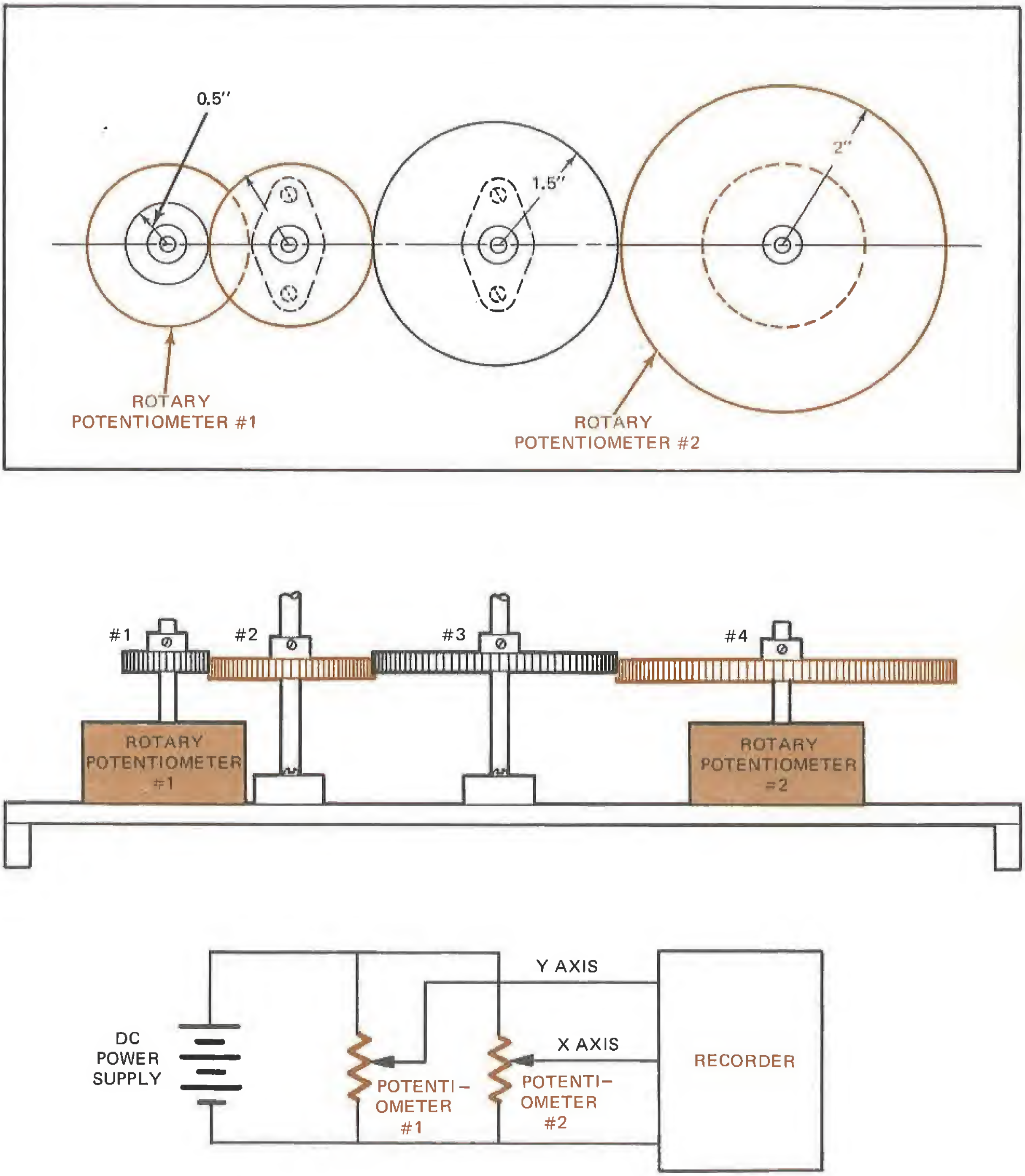


Fig. 13-9 Experimental Equipment

3. Turn the equipment on and obtain the displacement-displacement (angular) curve for gears 1 and 4.
4. Graphically differentiate this curve and find the velocity curve.
5. Using the velocity curve and equation 13.6, find the ratio of output torque to input torque.
6. Plot this ratio on a separate curve. Why does this curve have a constant value?
7. From the curve of step 4, what is the multiplication factor for output velocity? **(Don't neglect signs!)**
8. Connect potentiometer #1 into the X-axis and potentiometer #2 into the Y-axis. In effect, this will be the same as reversing the order of the gears.
9. Repeat steps 3 through 7, considering the 2 in. gear as input and the 0.5 in. gear as output.
10. Why do the slopes of the two displacement curves vary?
11. List as many possible values of velocity multiplication factors you can think of, combining the four gears in any desired order.

ANALYSIS GUIDE. Why did the displacement curves not come out as perfectly straight lines? You should consider such factors as play in the gears, friction in the potentiometers and linkage, and human error in connecting the linkage.

PROBLEMS

1. For two gears separated at their inside edges by 1.5 ft, we have the following information:

   $d_{in.} = 1$ ft $\qquad d_{out} = 1$ ft

   $\omega_{in.} = 200$ RPM $\qquad \omega_{out} = 375$ RPM

   There are to be three intermediate gears, and you may make any necessary assumptions, specifying them, to find the intermediate gear diameters.

2. For the gear train in figure 13-10, what is the ratio of output to input torque? What is the velocity ratio?

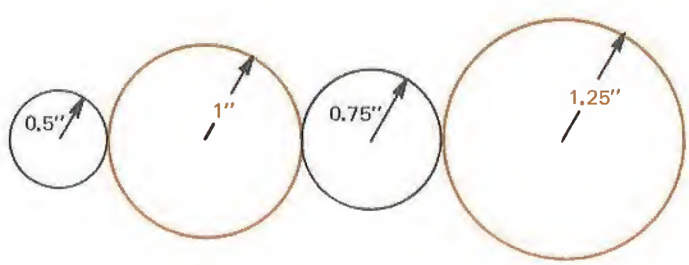


Fig. 13-10 Gear Train

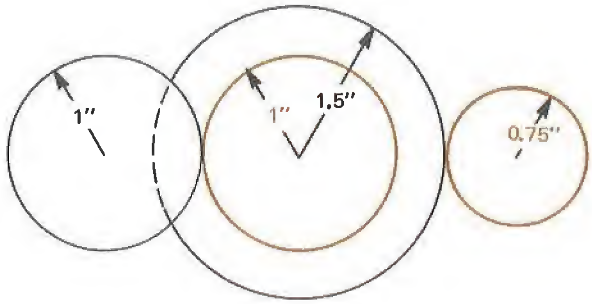


Fig. 13-11 Gear Train for Problem 3

3. For the gear train in figure 13-11, what is the ratio of output to input torque? What is the velocity ratio?
4. Sketch a gear train with a velocity multiplication factor of 3.93 with an otuput gear radius of 9 in..
5. Give a torque ratio of 9 to 1, what is the ratio of gear diameters?
6. Explain in your own words why the addition of simple intermediate gears does not affect the velocity ratio of the input and output gears.
7. Explain why the addition of intermediate gears, such as the 1 in. and 1.5 in. in figure 13-11, *will* affect the velocity ratio.

experiment 14 INTERMITTENT MOTION MECHANISMS

INTRODUCTION. The Geneva wheel is an excellent example of the intermittent motion class of mechanisms. In this experiment we will examine some of the characteristics of this mechanism.

DISCUSSION. There are many cases in which it is *necessary* to provide motion which is not continuous, but is repeated at regular intervals. There are several mechanisms which will provide this motion, among them a ratchet wheel, Geneva wheel and escapement wheel. In normal machine use, impact loading is undesirable, so velocity profiles should be similar to that of figure 14-1, if possible.

With the velocity profile of figure 14-1 for the driven member, there is no impact loading (instantaneous change in the value of acceleration). We also observe from figure 14-1 that the displacement curve is that of simple harmonic motion with lapses between each pulse and no negative displacement. The necessary displacement curve is illustrated by figure 14-2.

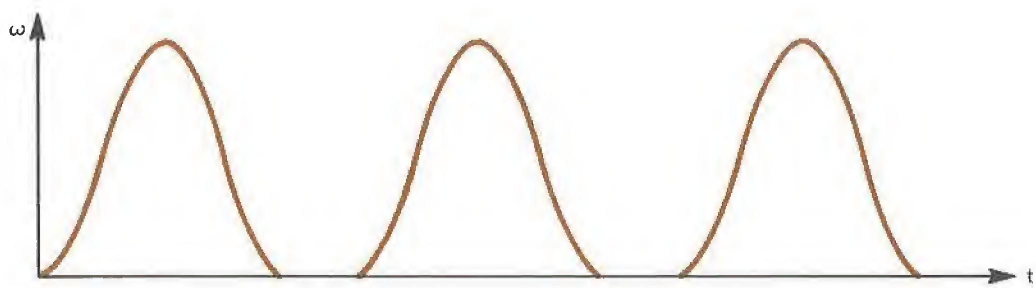


Fig. 14-1 Intermittent Motion Velocity Profile

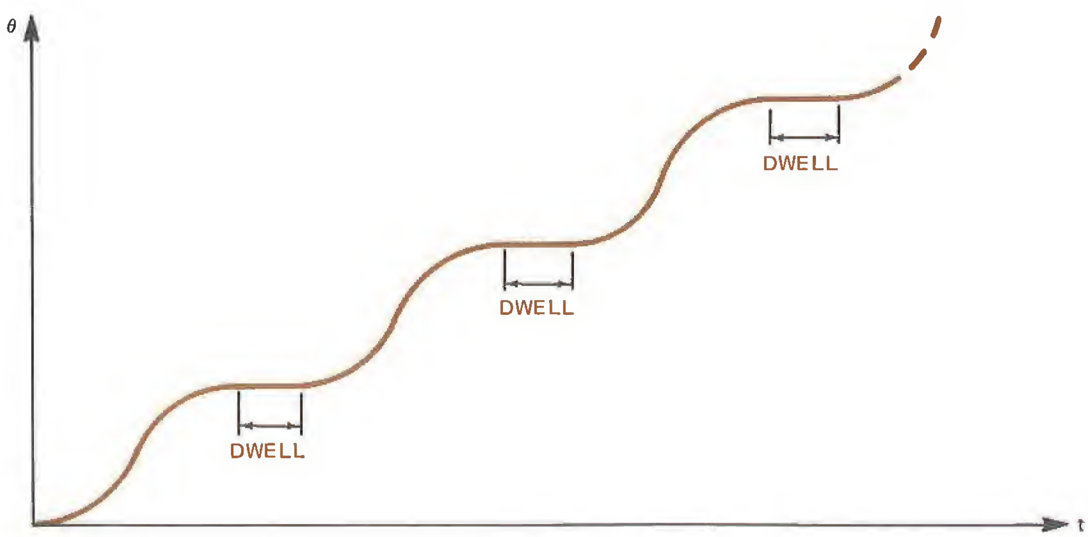


Fig. 14-2 Intermittent Motion Displacement Curve

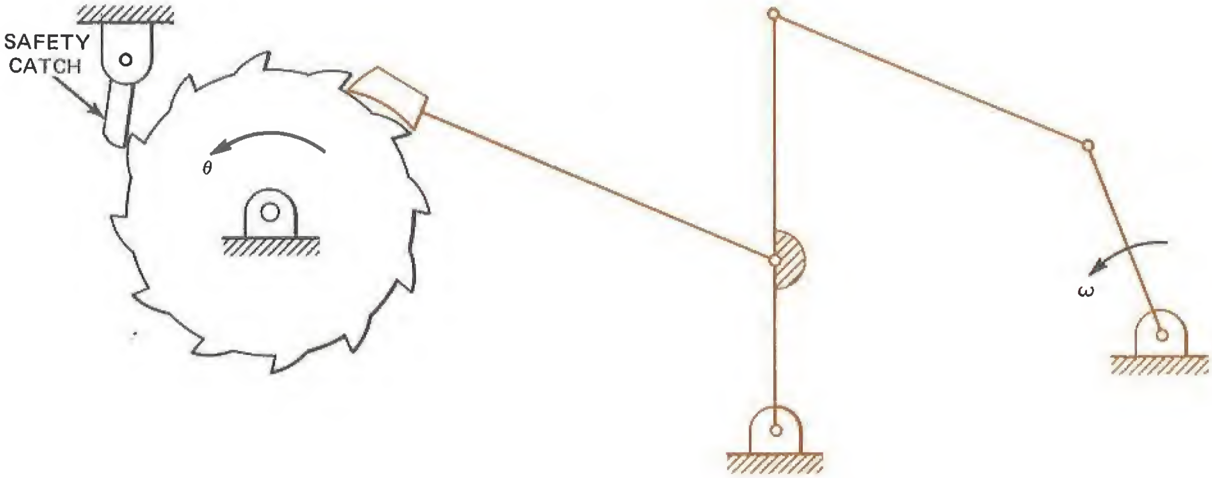


Fig. 14-3 Ratchet Intermittent Motion

A ratchet mechanism which will generate intermittent motion is illustrated in figure 14-3, with a safety catch to prevent back-spacing of the mechanism.

Figure 14-3 illustrates the basic idea of ratchet mechanisms. There are many possible linkages that will provide the same drive motion as the crank-rocker shown in the figure.

A ratchet mechanism can also be used which is capable of reversing the direction of rotation, as in figure 14-4. In figure 14-4, the direction of rotation of the ratchet may be reversed by changing the orientation of link A. If we keep the same sign convention, the characteristic curves for the reversed direction will be the *mirror images* of figure 14-1 and 14-2 on the *negative* θ and ω axes. This is illustrated in figure 14-5.

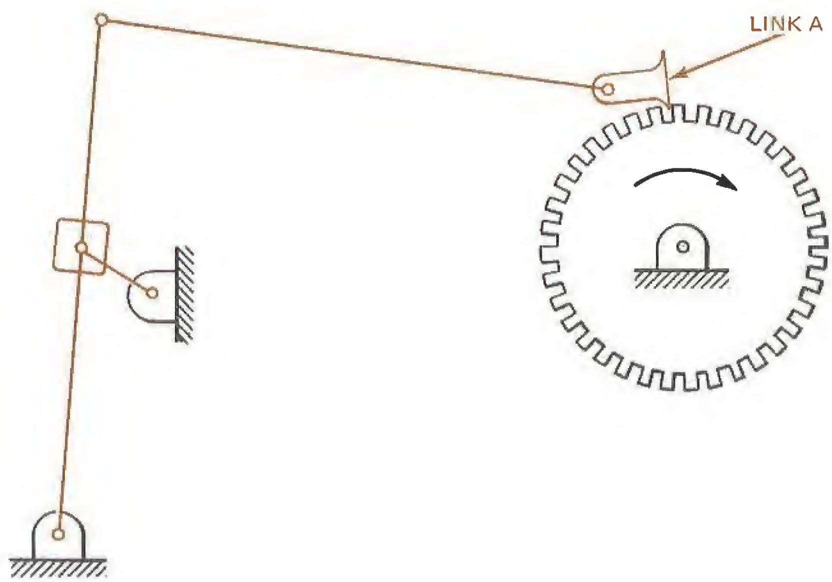


Fig. 14-4 Reversible Ratchet Mechanism

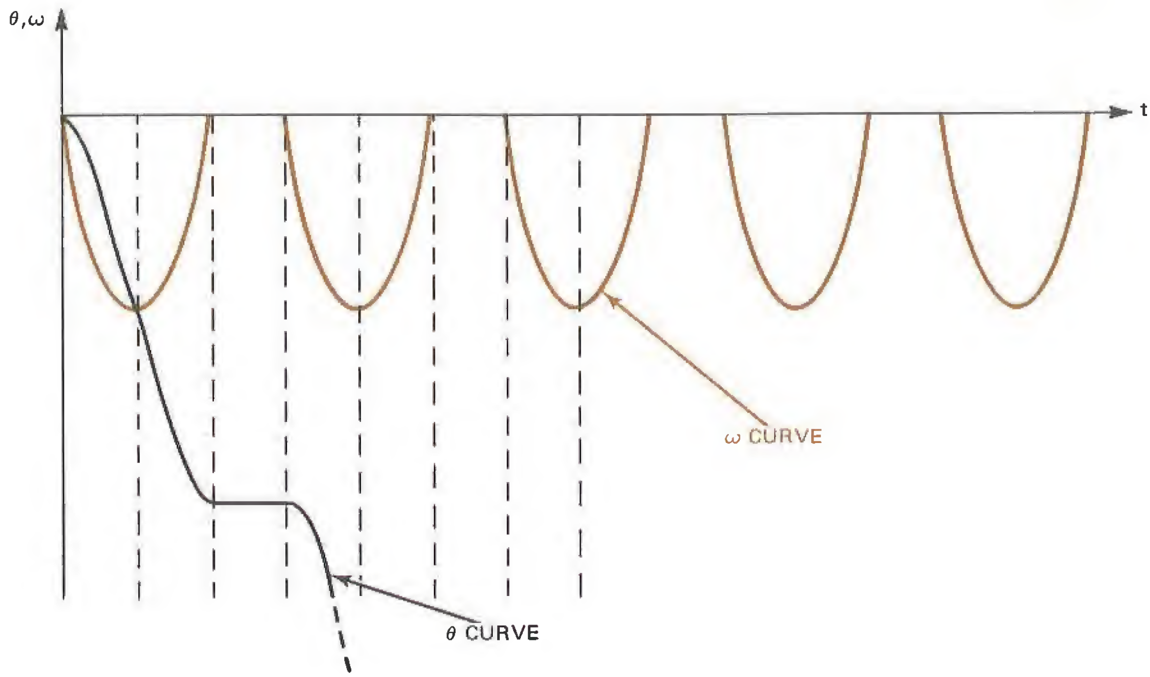


Fig. 14-5 Reversed Ratchet Curves (Mirror Image)

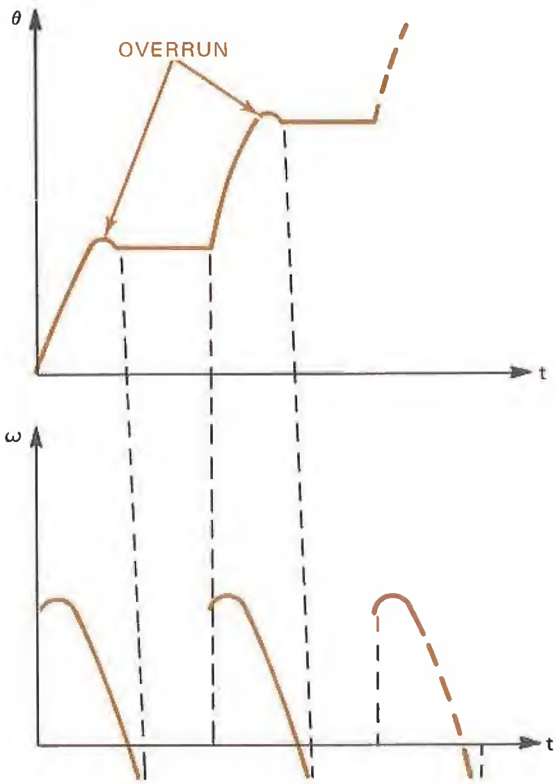


Fig. 14-6 Actual Curves

The characteristic curves for the ratchet mechanism were drawn with the assumption that there was no impact loading on the mechanism. A *realistic* set of curves would probably be similar to figure 14-6.

We observe from the velocity curve of figure 14-6 that the mechanism will have an impact loading as the velocity curve changes instantaneously with time. The acceleration will pulse at these points and, consequently, jerk (da/dt) will be infinite at these points. The *overrun* in the displacement curve will occur between the time the driver disengages and the safety catch stops the gear since an *actual* mechanism will have inertia effects and tolerance limits.

Ratchet mechanisms are frequently used in assembly-line work, since very small angular displacements can be achieved.

When larger values of angular displacement are desired, Geneva wheels may be used as illustrated by the 4-slot Geneva wheel in figure 14-7.

After the roller has entered the slot and the Geneva wheel is turning, the geometry is similar to the sketch in figure 14-8.

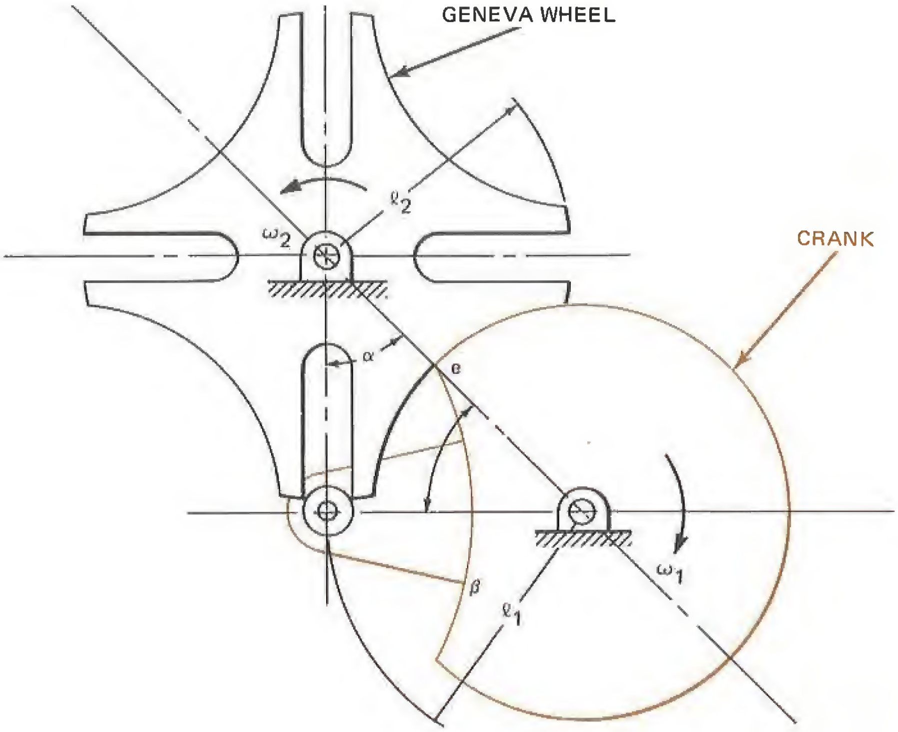


Fig. 14-7 Geneva Wheel

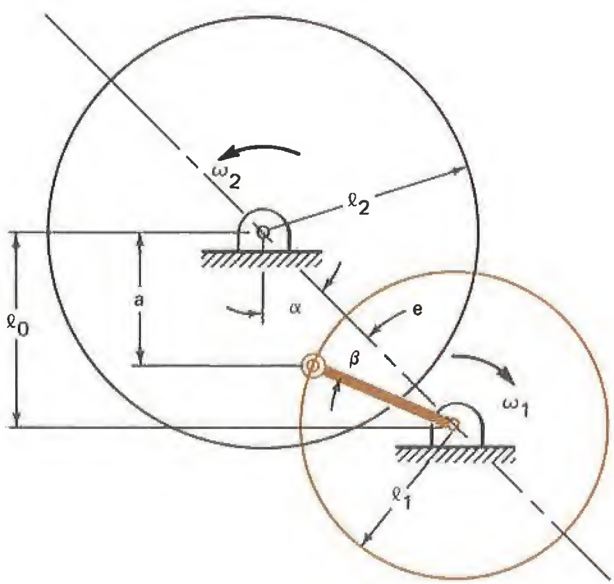


Fig. 14-8 Geneva Wheel Sketch

By inspection we observe that

$$(\ell_o - a) = \ell_1 \cos\beta$$

$$\boxed{a = \ell_o - \ell_1 \cos\beta} \qquad (14.1)$$

$$\beta_{max} = \frac{360^\circ}{2\,(\#\text{ of slots})} = \frac{180^\circ}{\#\text{ of slots}}$$

$$\boxed{\ell_o = \frac{\ell_1}{\sin\left(\dfrac{180^\circ}{\#\text{ of slots}}\right)}} \qquad (14.2)$$

From figure 14-8 we could also show that

$$\boxed{\alpha = \tan^{-1}\left(\frac{\sin\beta}{\left(\dfrac{e}{\ell_1}\right) - \cos\beta}\right)} \qquad (14.3)$$

and by differentiating equation 14.3, we can find the angular velocity of the wheel, for any β, as

$$\boxed{\omega_{wheel} = \frac{d\alpha}{dt} = \omega_1 \left[\frac{\left(\dfrac{e}{\ell_1}\right)\cos\beta - 1}{1 + \left(\dfrac{e}{\ell_1}\right)^2 - 2\left(\dfrac{e}{\ell_1}\right)\cos\beta}\right]} \qquad (14.4)$$

We observe from equation 14.4 that maximum wheel velocity corresponds to $\beta = 0$, giving

$$\boxed{\omega_{wheel/max} = \omega_1 \left(\frac{\ell_1}{e - \ell_1}\right)} \qquad (14.5)$$

By differentiating equation 14.4, we obtain the expression for angular acceleration of the wheel.

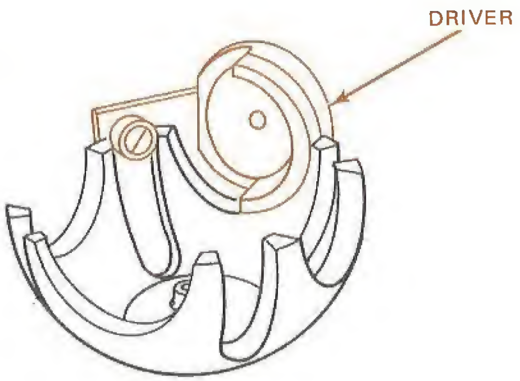
$$\boxed{\begin{aligned}\alpha_{wheel} &= \frac{d\omega_{wheel}}{dt} \\ &= \omega_1^2 \left[\frac{\left(\dfrac{e}{\ell_1}\right)\sin\beta\left(1 - \dfrac{e^2}{\ell_1^2}\right)}{\left(1 + \left(\dfrac{e}{\ell_1}\right)^2 - 2\left(\dfrac{e}{\ell_1}\right)\cos\beta\right)^2}\right]\end{aligned}} \qquad (14.6)$$

By making the appropriate substitutions and manipulations, we can show that maximum angular acceleration, $\alpha_{wheel/max}$, occurs at

$$\boxed{\beta = \cos^{-1}\left[\pm\sqrt{\left[\frac{1 + \left(\dfrac{e}{\ell_1}\right)^2}{4e/\ell_1}\right]^2 + 2} - \frac{1 + \left(\dfrac{e}{\ell_1}\right)^2}{4e/\ell_1}\right]} \qquad (14.7)$$

The other two types of Geneva wheel are shown in figure 14-9. The type in figure 14-8 is called an *external* Geneva wheel, and the two types in figure 14-9 are internal and spherical, as their shapes imply.

Escapement mechanisms are commonly used when small angular displacements with a very small amount of play in the mechanism are desired. An example of this is shown in figure 14-10 and is a common mechanism in wall clocks. This mechanism could be adapted to wrist watches by powering the pendulum link with a secondary linkage driven by the mainspring. When P_1 is starting upward, the escapement wheel moves clockwise a small amount since the link at P_1 is slanted on one edge. Then P_2 goes down into the start of the next slot and prevents excessive motion. This cycle is repeated over and over, and the wheel moves in increments.



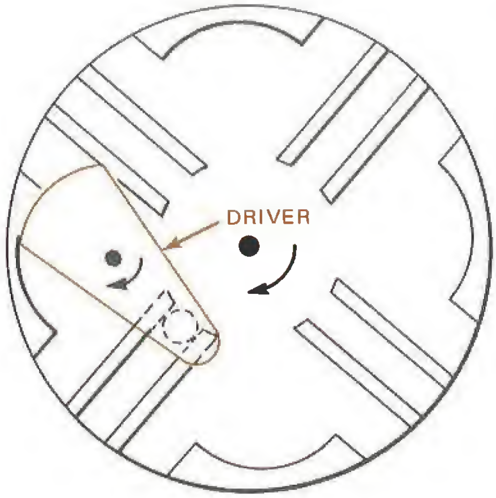


Fig. 14-9 Spherical and Internal Geneva Wheels

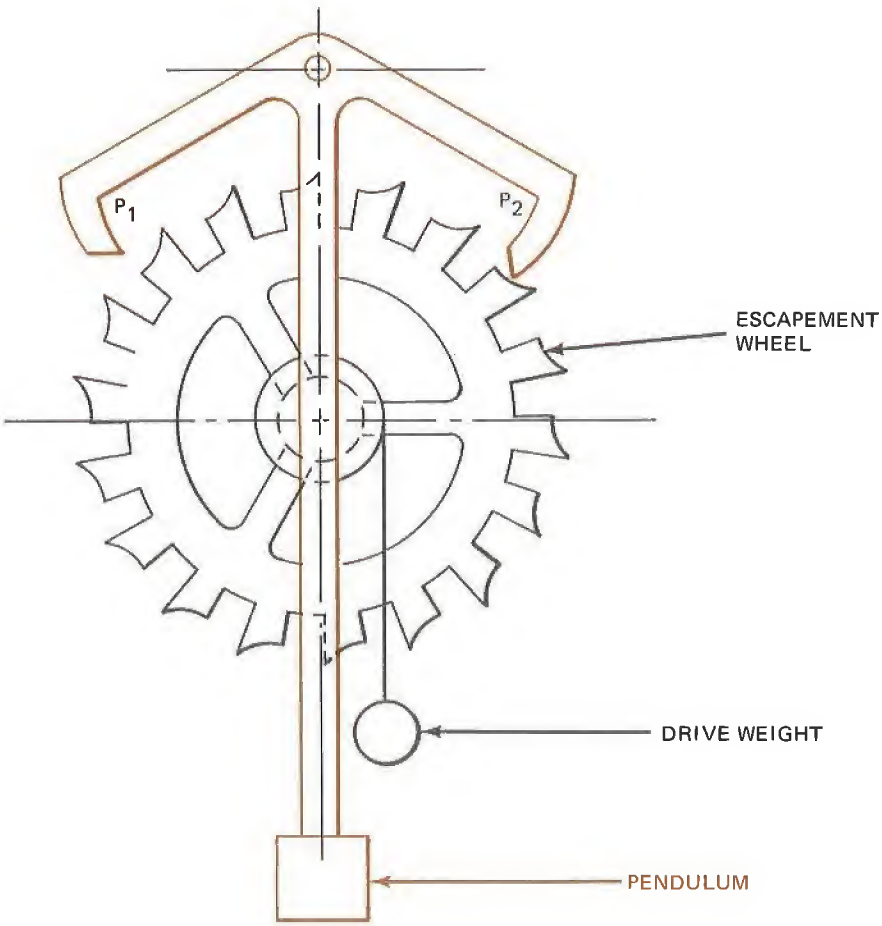


Fig. 14-10 Escapement Mechanism

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with short spacers
2 Rotary potentiometers
6 Collars
2 Couplings
4 Bearing holders with bearings
1 Geneva wheel, d = 3 in., 6 slots
1 Geneva wheel, d = 4 in., 6 slots
1 Driver for 3 in. wheel
1 DC power supply
1 X-Y or strip-chart recorder
2 1/4 X 4 in shafts

PROCEDURE

1. Inspect all experimental components to be sure they are undamaged.
2. Connect the apparatus as illustrated by figure 14-11.

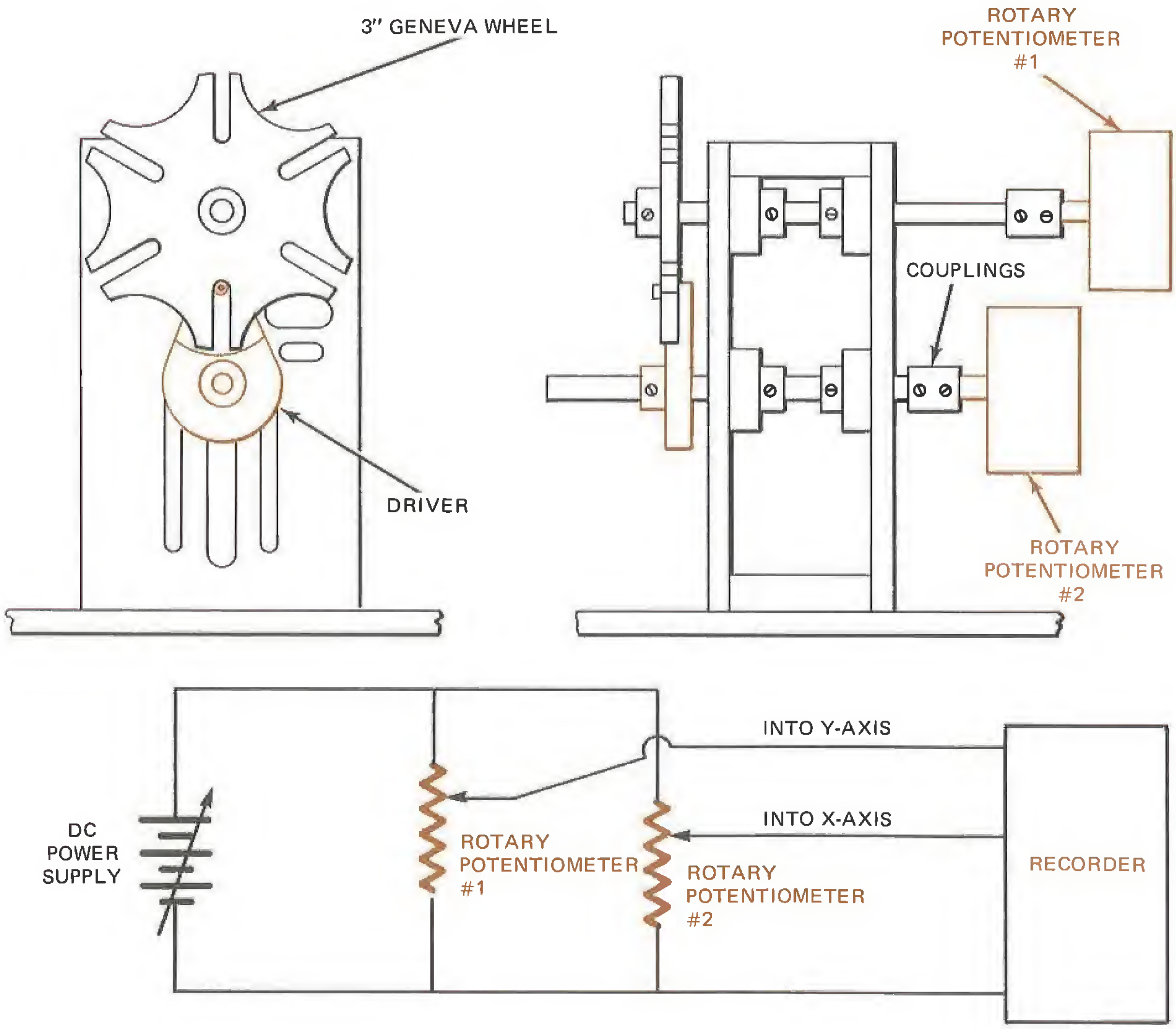


Fig. 14-11 Experimental Equipment

3. Operate the mechanism for 360° of the Geneva wheel to obtain the displacement-displacement plot for the linkage. **NOTE: It will be necessary to reset the zero point for potentiometer #2 each cycle of the driver.**
4. Visually compare this curve with that of figure 14-2.
5. Graphically differentiate this curve to obtain the velocity and acceleration curves.
6. Using equation 14.4, *calculate* the value of ω_{wheel} at $\beta = 30°$.
7. Exchange the Geneva wheel for an 8-slot, 4-in. diameter one and make any necessary adjustments in the equipment to get it to operate properly, without changing ℓ_1.
8. Repeat steps 3 through 6.
9. Visually compare the curves.
10. How do the two curves vary?

ANALYSIS GUIDE. Explain in your own words how and why the two sets of curves vary. You should consider such factors as human error in aligning the mechanism, play in the linkage, and calibration errors.

PROBLEMS

1. If a Geneva mechanism is designed so that the driver is not *normal* to the slot as it enters it, what will the jerk (da/dt) curve look like?
2. Would the mechanism in problem 1 be suitable for high RPM work? Expalin.
3. Given a Geneva mechanism with ℓ_1 = 3 in., ω_1 = 100 RPM, $\beta_{max} = 45°$, calculate ℓ_0. For $\beta = 15°$, find a.
4. For the Geneva mechanism in problem 3, what should ℓ_2 be to prevent impact loading? (HINT: Find ℓ_2 such that driver enters and leaves slots at 90°.)
5. In problem 4 find ω_{wheel} for $\beta = 45°$.
6. Draw a rough sketch of the velocity curve for the Geneva mechanism in problem 3-5.
7. If the driver enters the slot with a 30° angle between them, what will the velocity curve look like?

experiment 15 SUMMARY OF TECHNIQUES

INTRODUCTION. In these experiments we have encountered a number of analytical techniques. Some of these methods are quite important. In this exercise we will summarize the main graphic analysis techniques used so far.

DISCUSSION. Perhaps the best way we can approach this summary is to list each topic we have covered and provide a brief sketch of each. The first two exercises on the basic drafting methods and equipment calibration will be omitted since they have formed an integral part of every experiment.

Link Point Curves may be found for linkages which are constrained to move in a particular path at all times. We recall that the procedure is to draw the *known* paths of motion, and by using the known link lnegths, the displacement curve for a link or a point may be found. This procedure is illustrated in figure 15-1.

Once the displacement curve is found, the velocity and acceleration curves may be found by graphic differentiation as shown in figure 15-2.

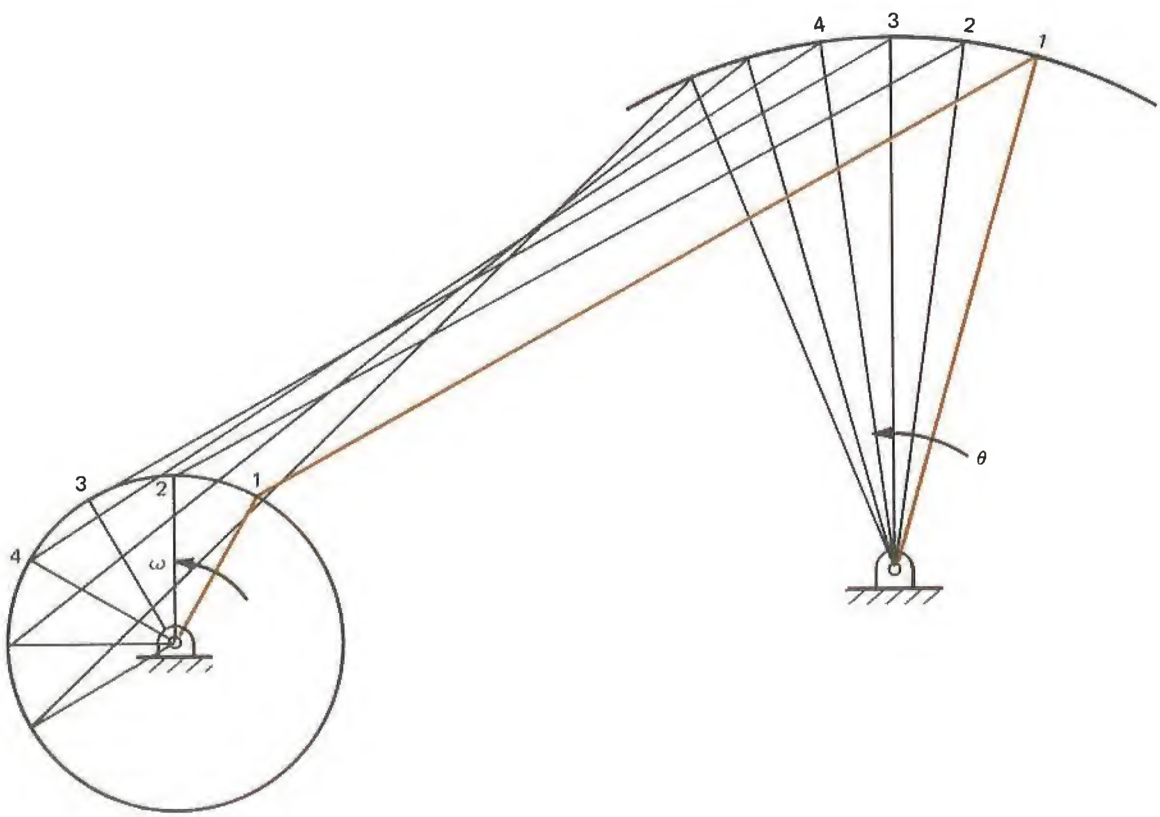


Fig. 15-1 Four-Bar Linkage

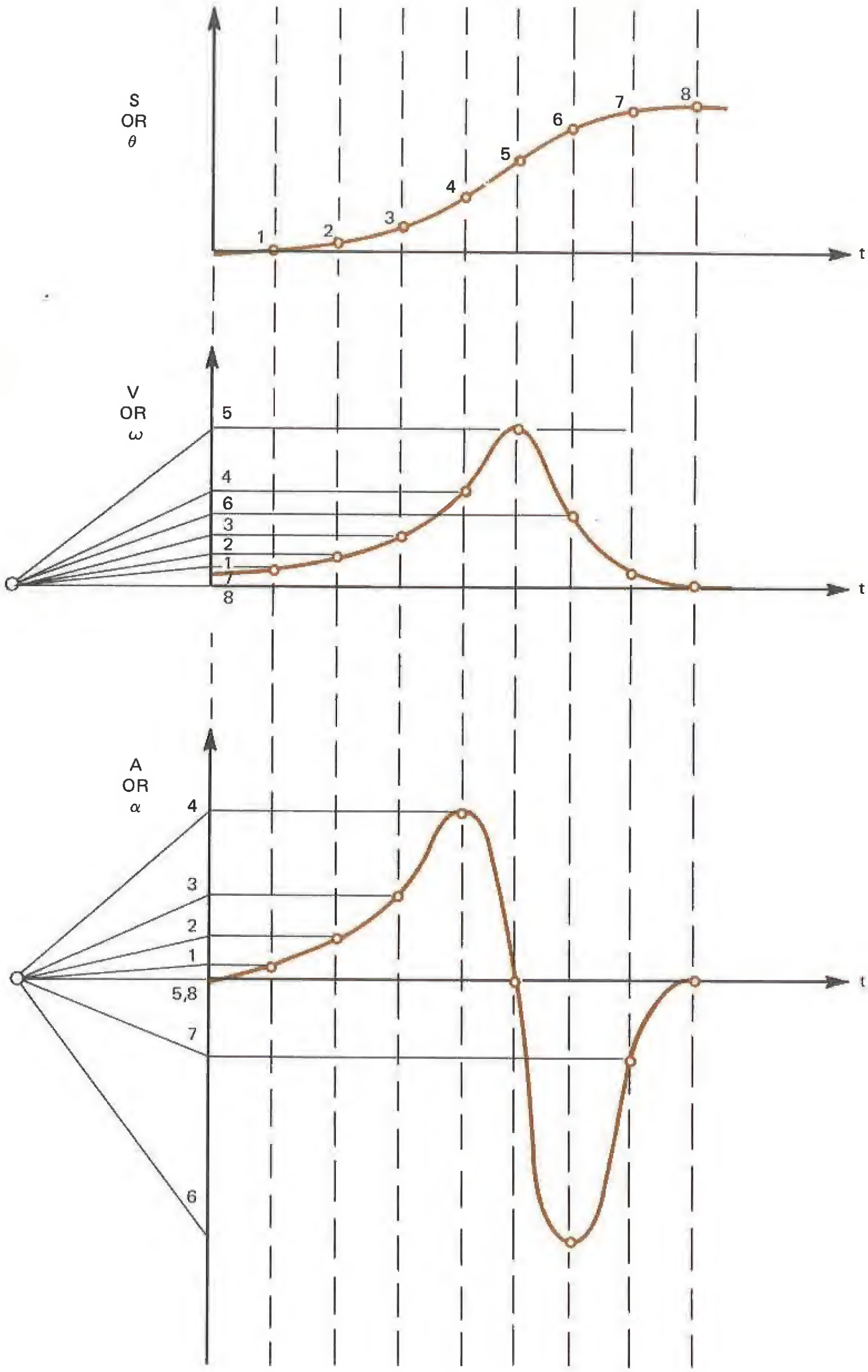
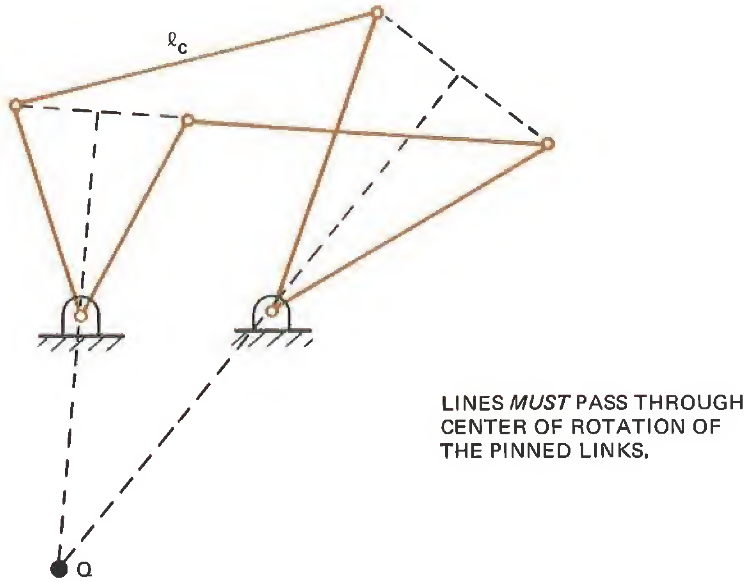
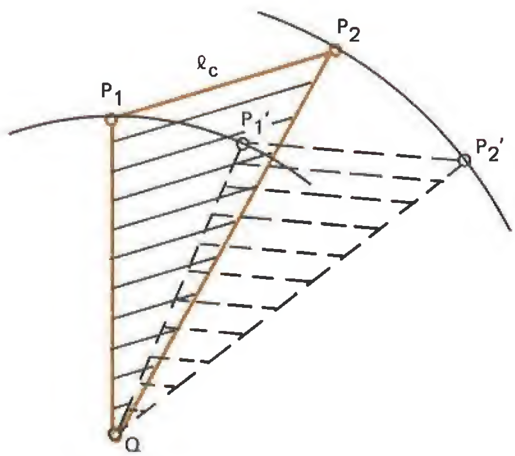


Fig. 15-2 Graphic Differentiation



A. POINT OF INTERSECTION

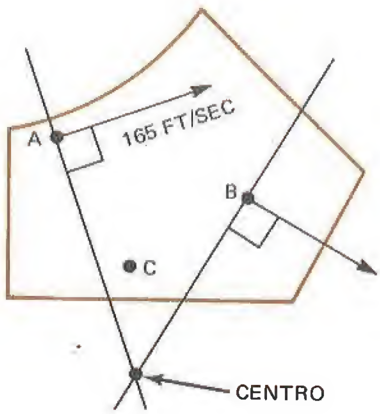


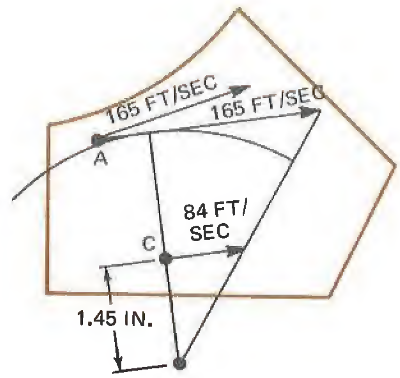
B. PURE ROTATION

Fig. 15-3 Instantaneous Centers

Instantaneous Centers are found by considering all motion to be instantaneous *rotation.* When perpendicular bisectors are erected on the line of motion of a link, their point of intersection is the center of rotation of the link as shown in figure 15-3.

We also used the line of proportionality for velocity and acceleration for rotating rigid bodies. By combining this concept and that of pure rotation, we may find the instantaneous velocity or acceleration of any point on the body as shown in figure 15-4,





$$V_c = 84\ \frac{\text{FT}}{\text{SEC}}$$

$$V_c = R_c\omega_c$$

$$\omega_c = \frac{V_C}{R_c}$$

$$\omega_c = \frac{84\ \text{FT/SEC}}{(1.45\ \text{IN.})\ (\text{FT}/12\ \text{IN.})}$$

$$\boxed{\omega_c = 695\ \frac{\text{RAD}}{\text{SEC}}}$$

Fig. 15-4 Velocity Measurement

for the unknown velocity for point C.

Slider-Crank Characteristics are normally similar to those of simple harmonic motion, as long as $\ell_c >> \ell_1$, as illustrated by the set of curves in figure 15-5.

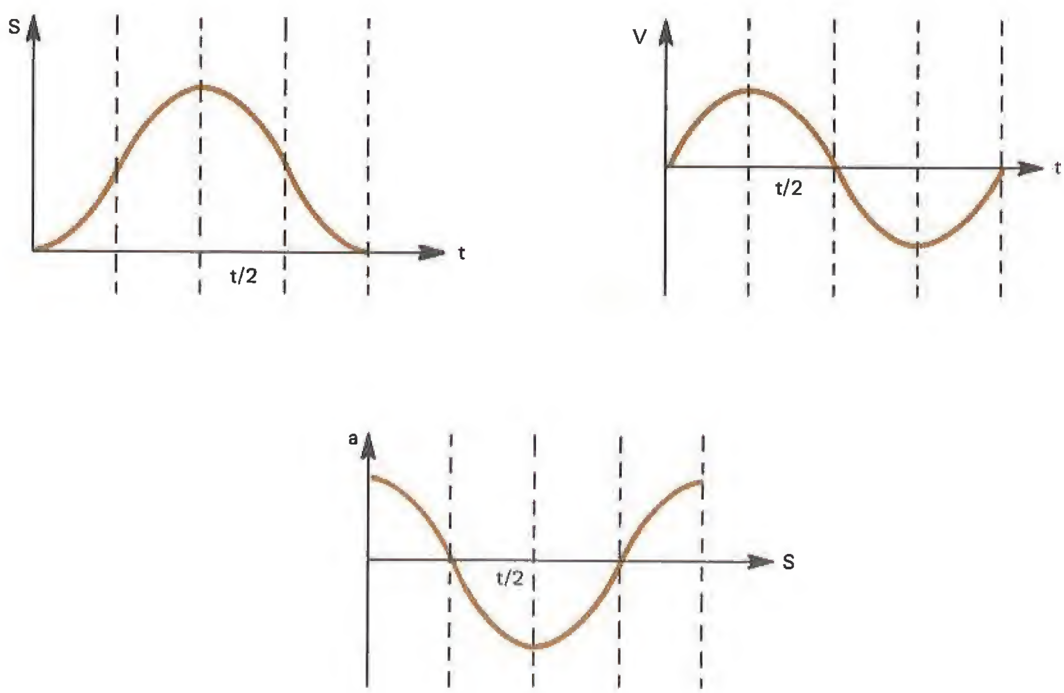
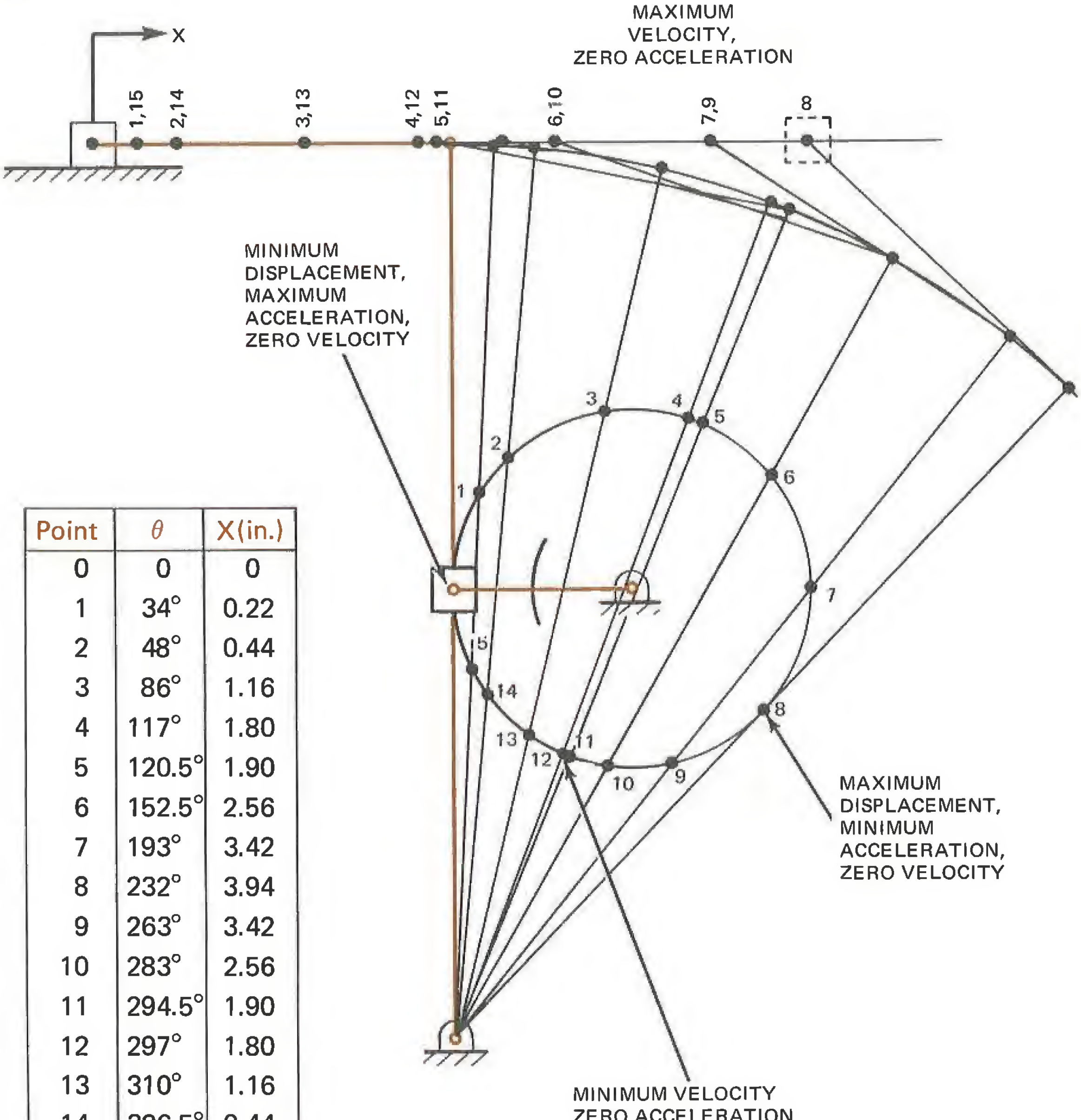


Fig. 15-5 Slider Crank Motion Curves

If we combine the slider-crank linkage with a quick return mechanism, giving the linkage in figure 15-6, then the characteristic curves change and become similar to those of figure 15-7.



Point	θ	X(in.)
0	0	0
1	34°	0.22
2	48°	0.44
3	86°	1.16
4	117°	1.80
5	120.5°	1.90
6	152.5°	2.56
7	193°	3.42
8	232°	3.94
9	263°	3.42
10	283°	2.56
11	294.5°	1.90
12	297°	1.80
13	310°	1.16
14	326.5°	0.44
15	335°	0.22

Fig. 15-6 Finding Displacement

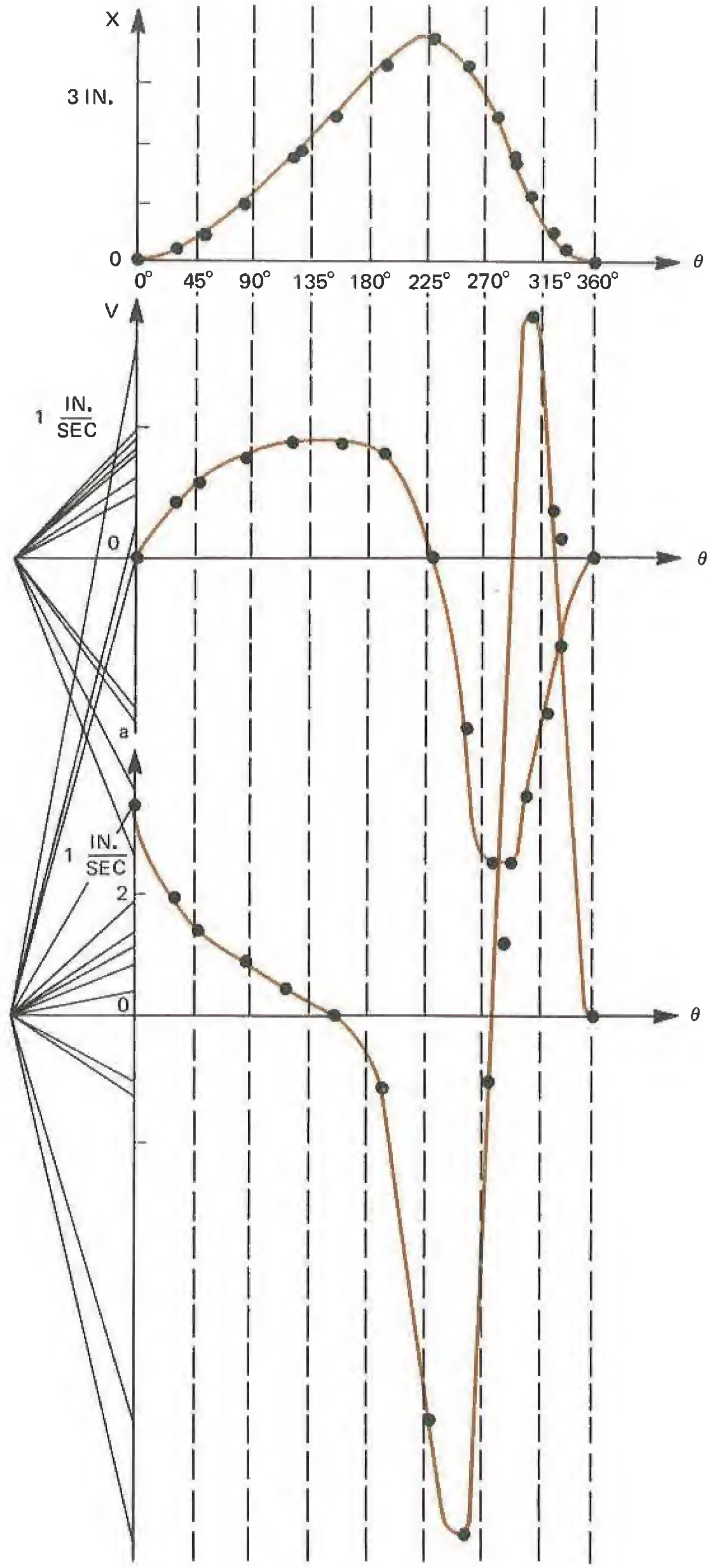


Fig. 15-7 Motion Curves

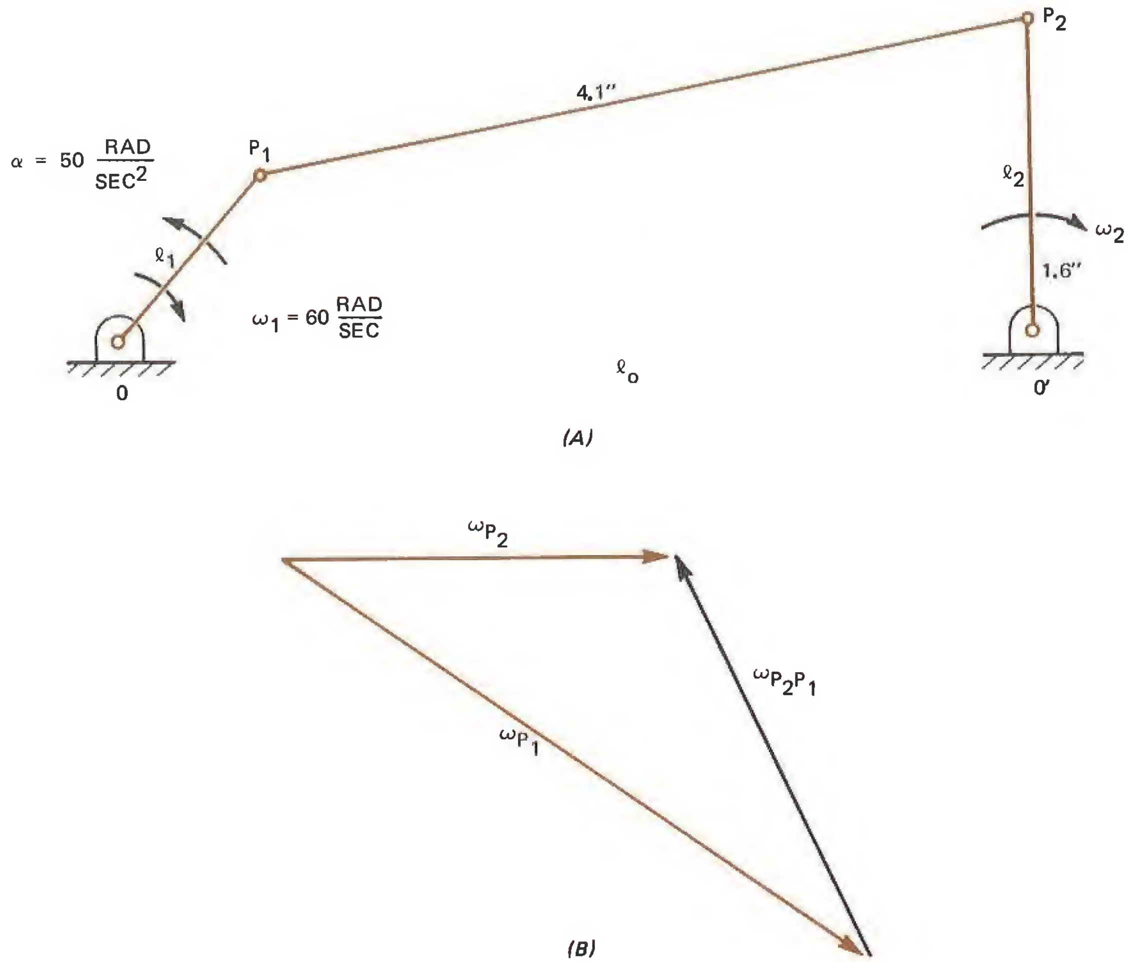


Fig. 15-8 (A) A Four-Bar Linkage
(B) Velocity Polygon

Velocity and acceleration polygons are often used when we wish to find the velocity or acceleration at only a few points. We recall that the velocity at all points is *normal to* the radius line to the center of rotation, and we use this fact to draw the velocity polygon, as in figure 15-8.

Straight-line mechanisms normally have either one point or one link which travels in a straight line. There are many different kinds of linkages which will generate straight-line motion, some of which are illustrated by figure 15-9.

Toggle Mechanisms are commonly used to amplify force at the expense of displacement. In certain portions of their cycle, extremely large forces may be generated. The governing equation for this is

$$\boxed{\frac{F_{out}}{F_{in}} = \frac{X_{in}}{X_{out}}}$$

This type of mechanism is illustrated by the rock-crusher in figure 15-10.

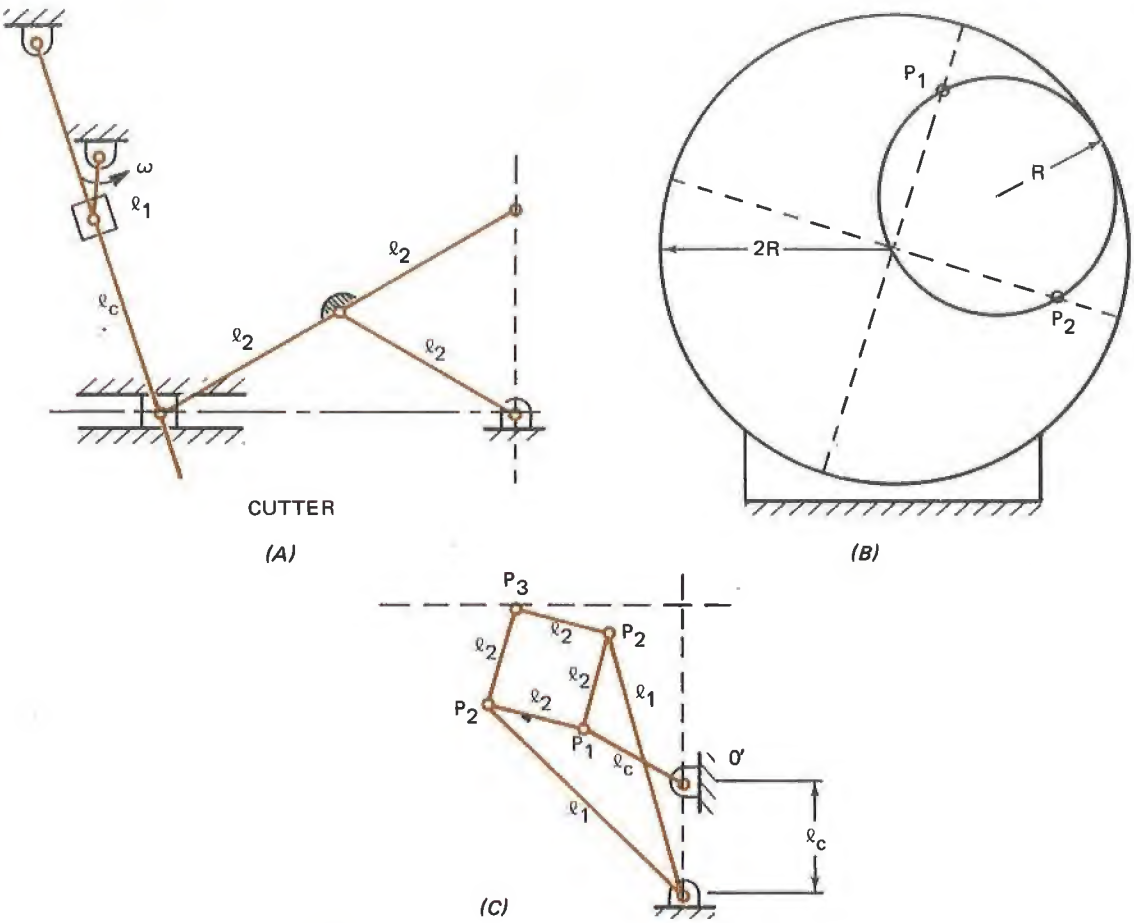


Fig. 15-9 (A) Quick-Return Straight-Line Motion
(B) Straight-Line Generator (C) Straight-Line Generator

Mechanical Computing Mechanisms have several uses: among these are mechanical computers, speedometers, and RPM gages. There are many different types of mechanical computing mechanisms, and the relationship may often be changed by just changing the output scale factor. Some of the more common types of mechanical computing mechanisms are shown in figure 15-11.

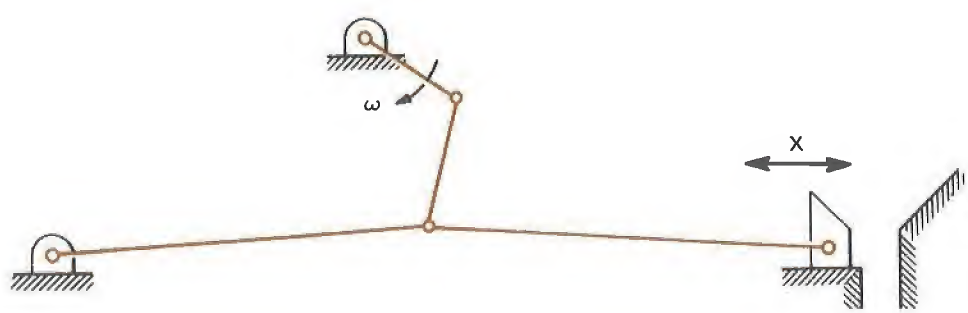
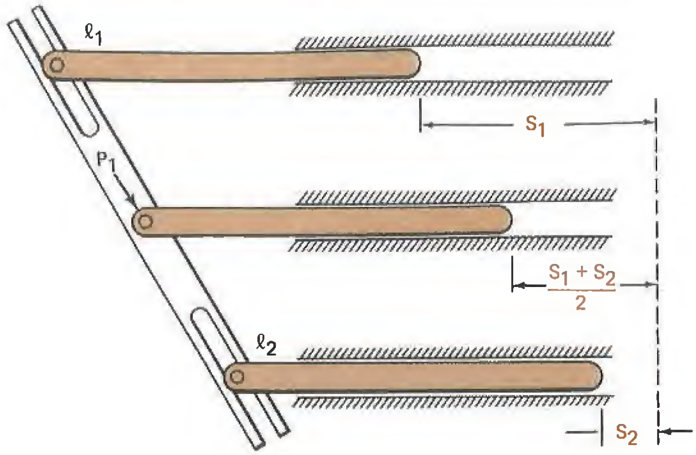
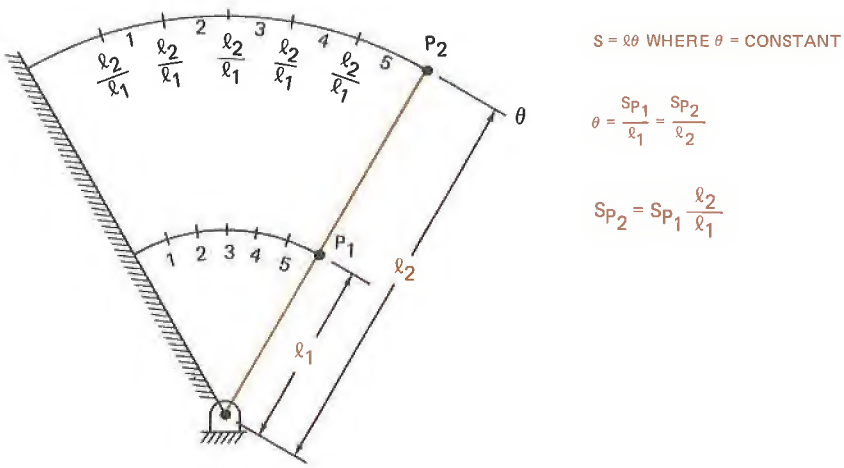


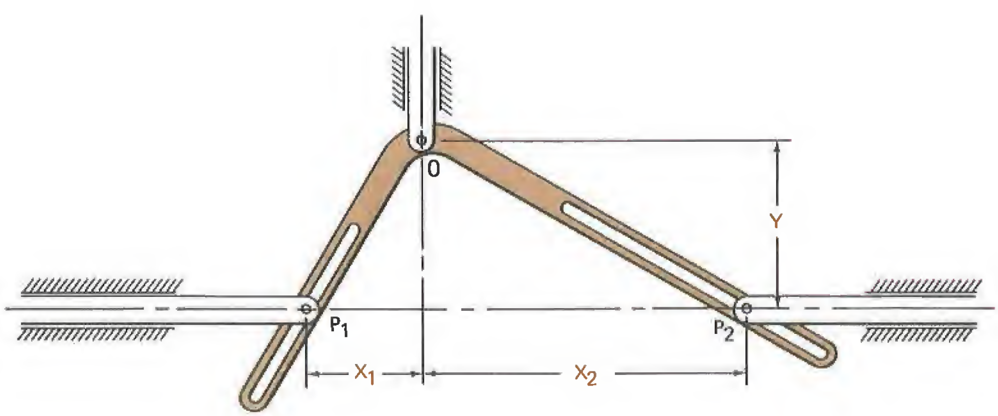
Fig. 15-10 Rock Crusher with Secondary Linkage



(A)



(B)



(C)

Fig. 15-11 (A) Averaging Mechanism
(B) Multiplication Mechanism
(C) Product Computing Linkage

Cams with single followers are often used when space limitations prevent using a more complex linkage, and when repititious motion is desired. The three major types of cams are the disc, cylindrical and translational cams, as illustrated in figure 15-12 with the disc and translational cam displacement curves also shown.

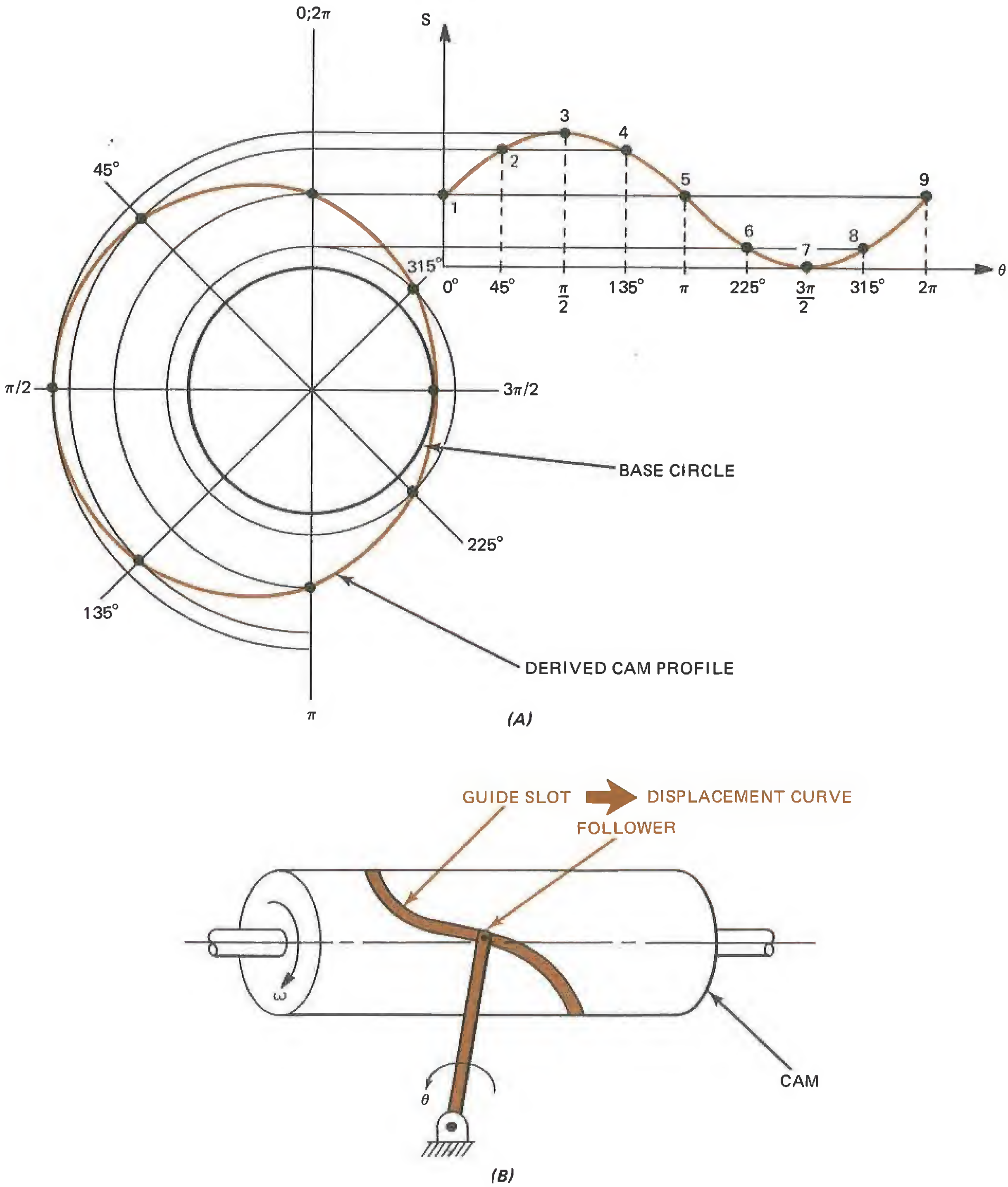
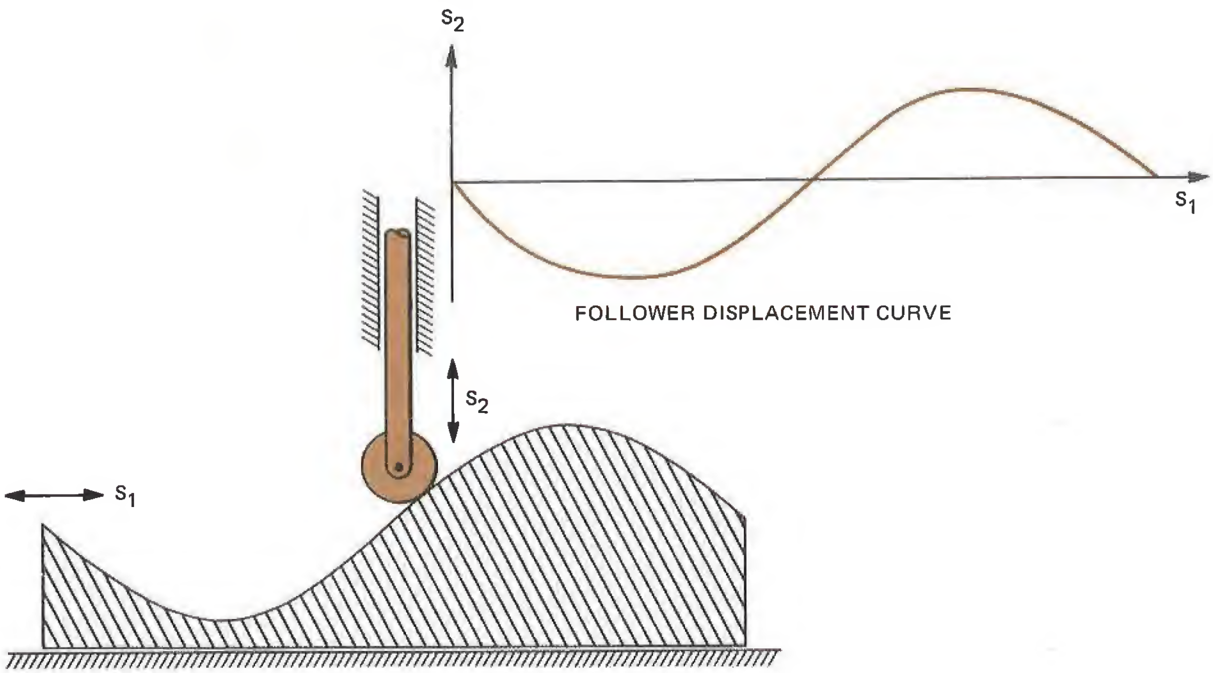


Fig. 15-12 (A) Finding Disc Cam Profile (B) Cylindrical Cam



(C)

Fig. 15-12 (C) Translational Cam (Cont'd)

Cams with double followers are often used to amplify the output displacement as illustrated by the double-follower cam in figure 15-13.

The result of amplifying a displacement curve is illustrated by the example of figure 15-14.

Complex motion cams are unusual in that they can be combined with linkages to generate two-dimensional motion as in the example of figure 15-15.

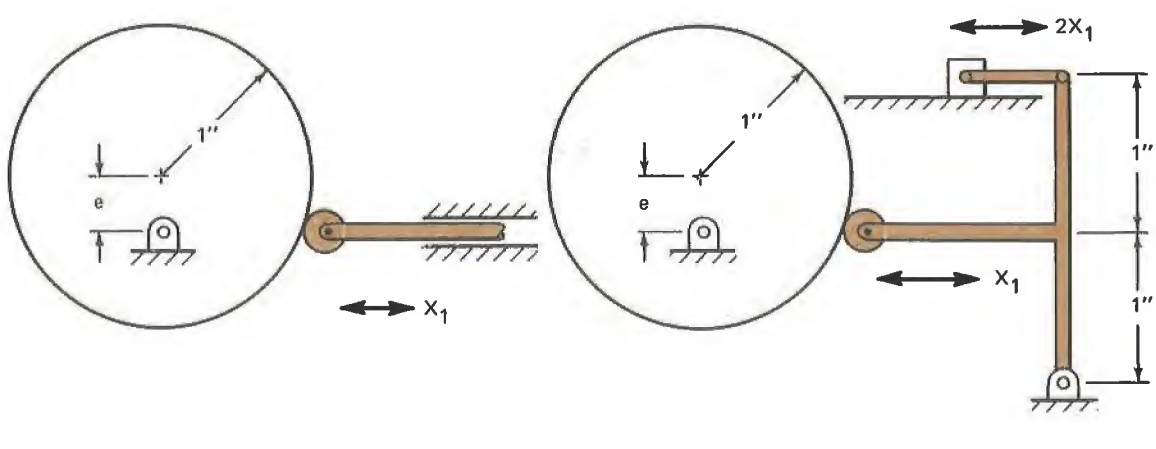


Fig. 15-13 Amplified Displacement

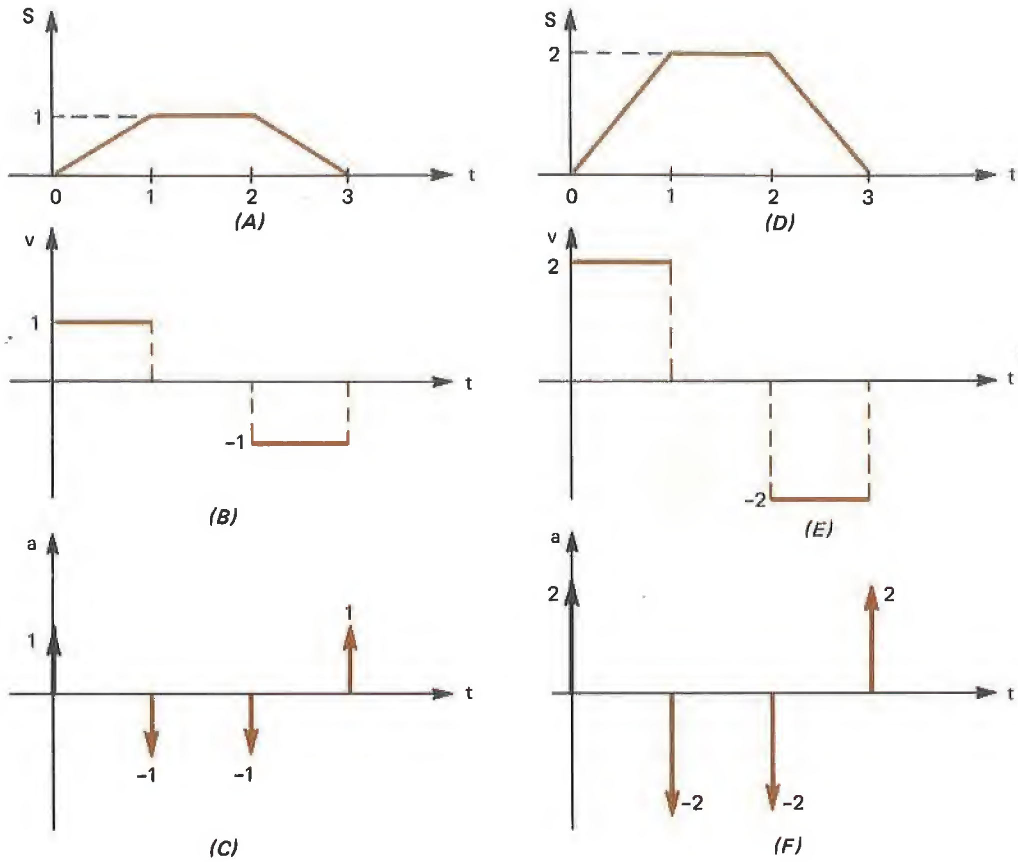


Fig. 15-14 Amplified Displacement Curve
(A), (B), (C) Original
(D), (E), (F) Amplified

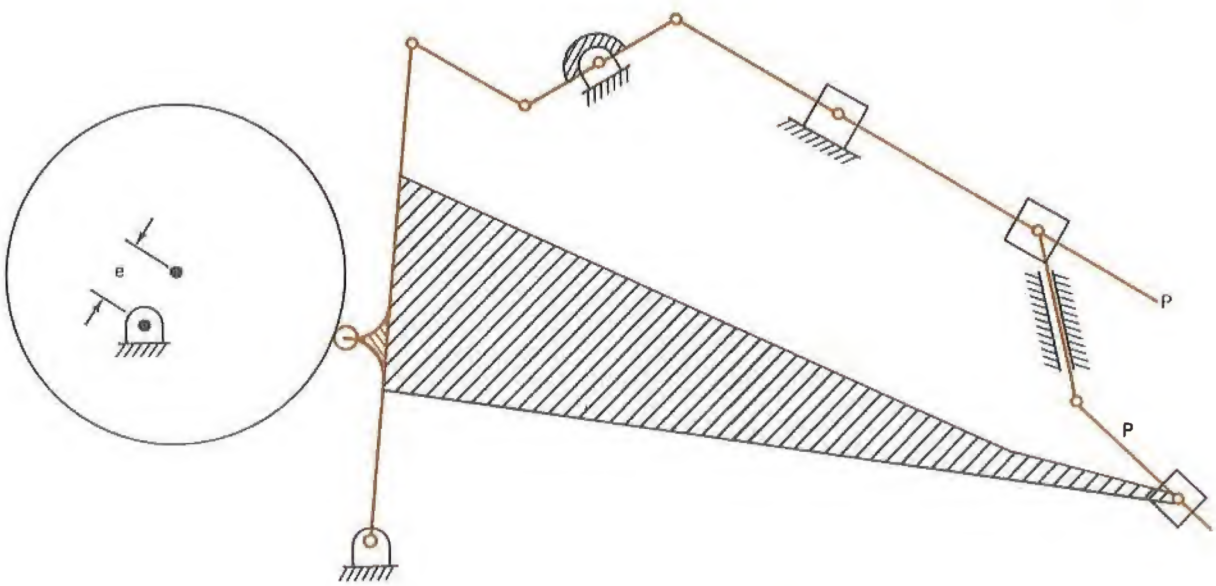


Fig. 15-15 Two-Dimensional Motion

The cam profile *or* the displacement curve for double cams may be found by using the method illustrated in figure 15-16.

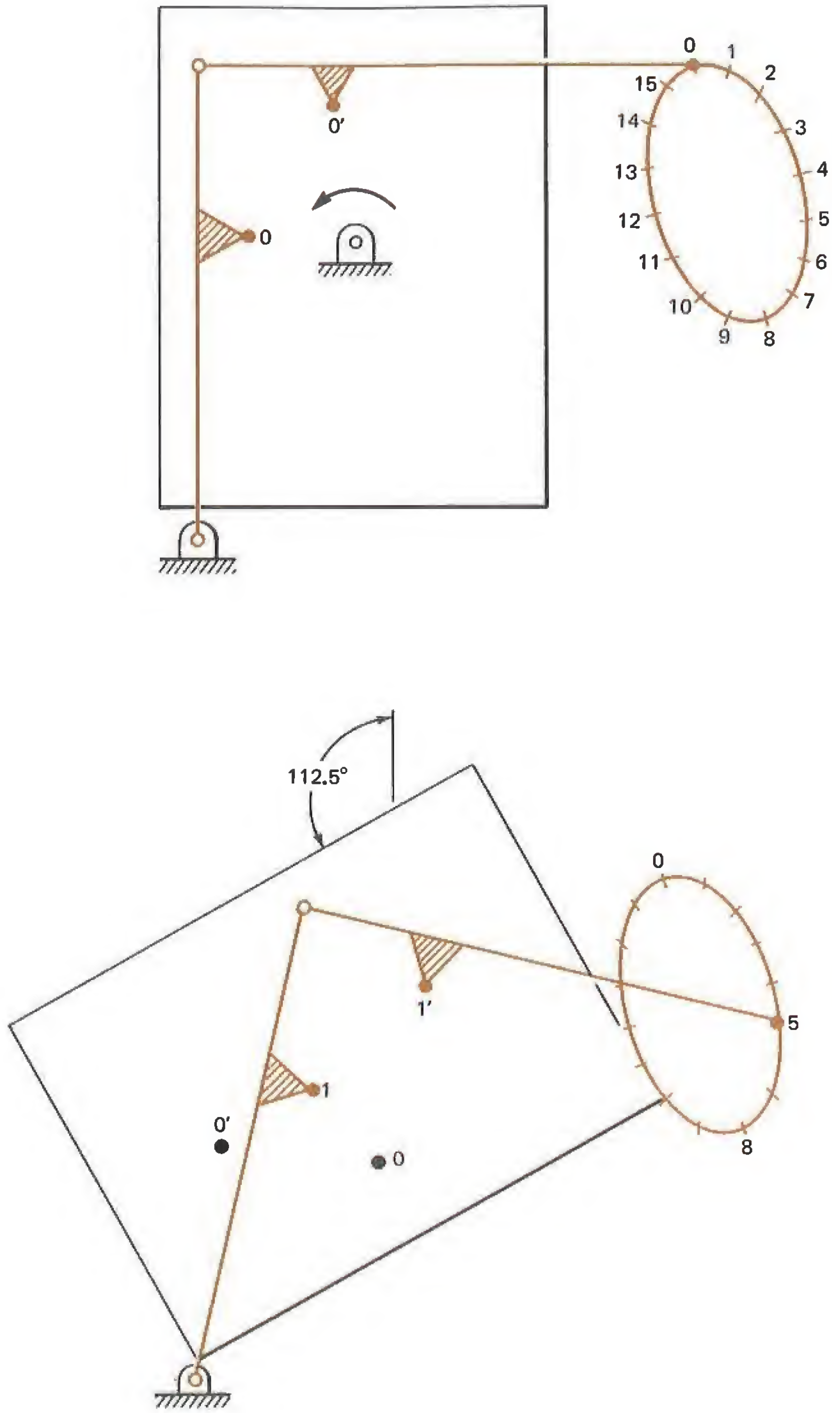
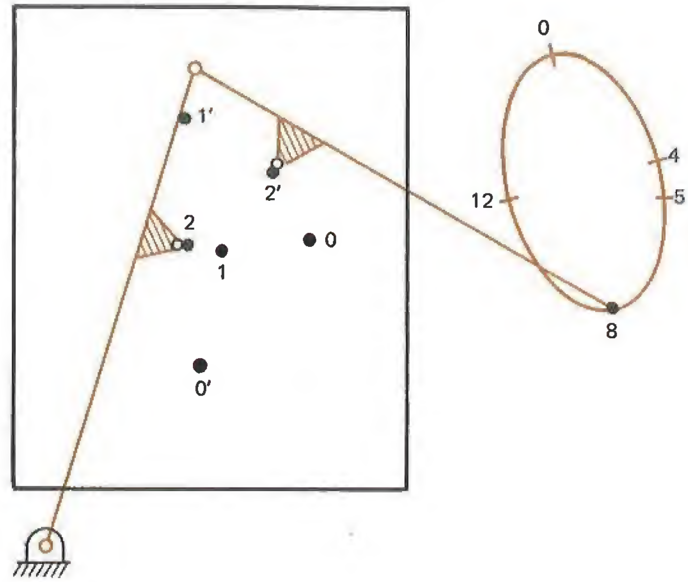


Fig. 15-16 Two Dimensional Motion Cam Profile



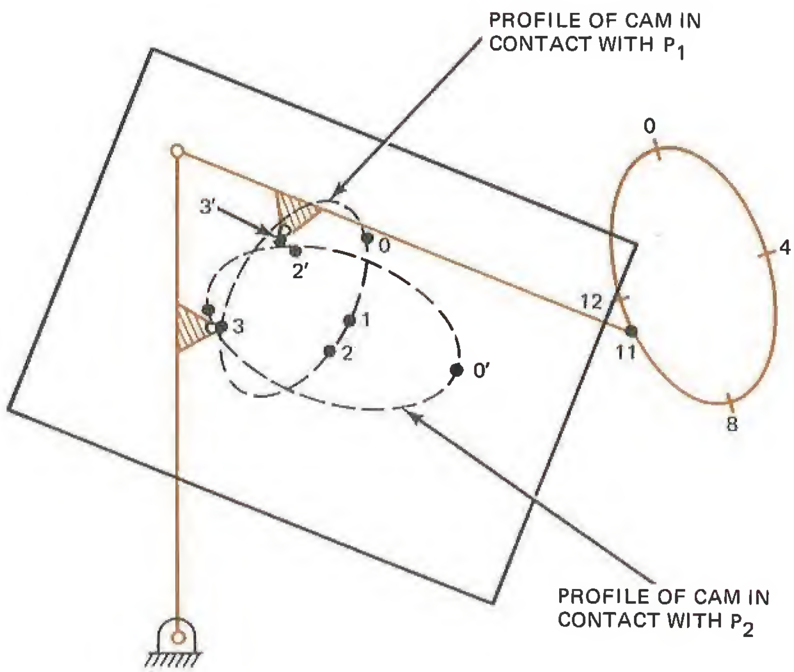


Fig. 15-16 Two Dimensional Motion Cam Profile (Cont'd)

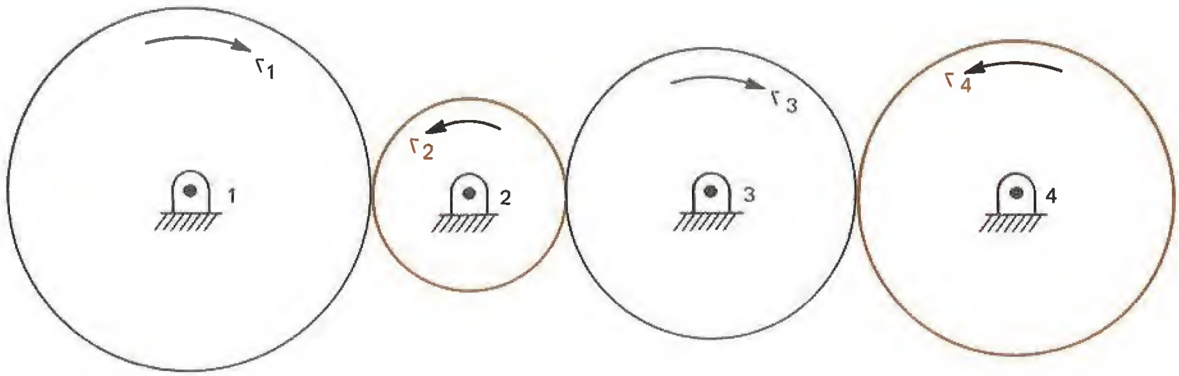


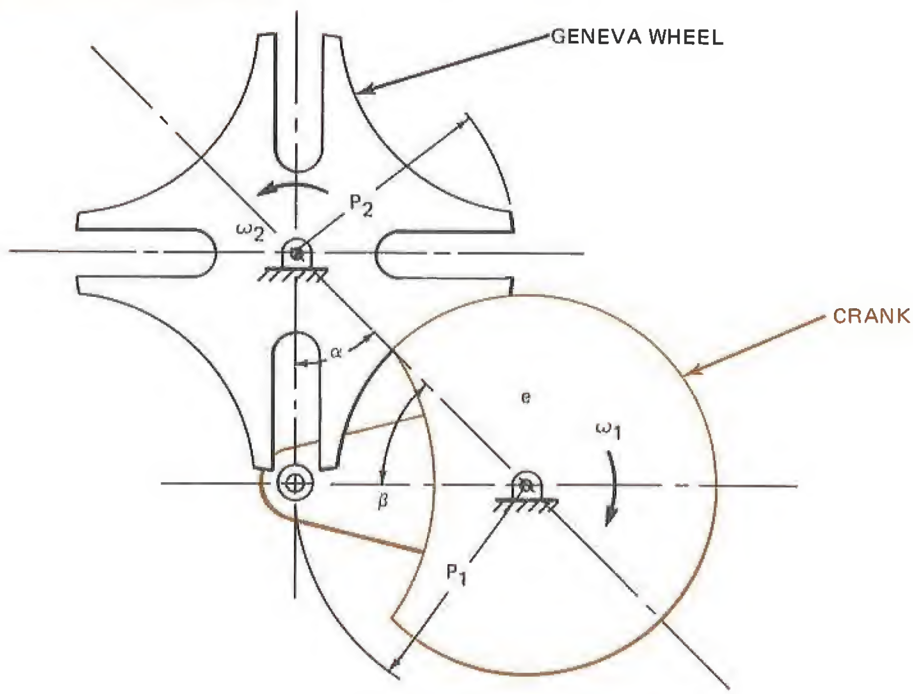
Fig. 15-17 Gear Train Torque Amplification

Velocity multiplication with gears is often used when the output ω needs to have a different value than the input ω, and the distance between input and output is not too large. The relationship between the torque ratio and velocity ratio is illustrated by figure 15-17 and equation 15.1.

$$\boxed{\frac{\tau_4}{\tau_1} = \frac{\omega_1}{\omega_2}\ \frac{\omega_2}{\omega_3}\ \frac{\omega_3}{\omega_4} = \frac{\omega_1}{\omega_4} = \frac{r_4}{-r_1}} \quad (15.1)$$

$$\tau_4 = \tau_1 \frac{\omega_1}{\omega_4} = \tau_1 \frac{r_4}{r_1}$$

Intermittent motion mechanisms are exemplified by ratchet, Geneva and escapement wheels. In normal practice, these are designed to eliminate as much impact loading as possible. These three types of intermittent motion generating mechanisms are illustrated in figure 15-18.



(A)

Fig. 15-18 (A) Geneva Wheel

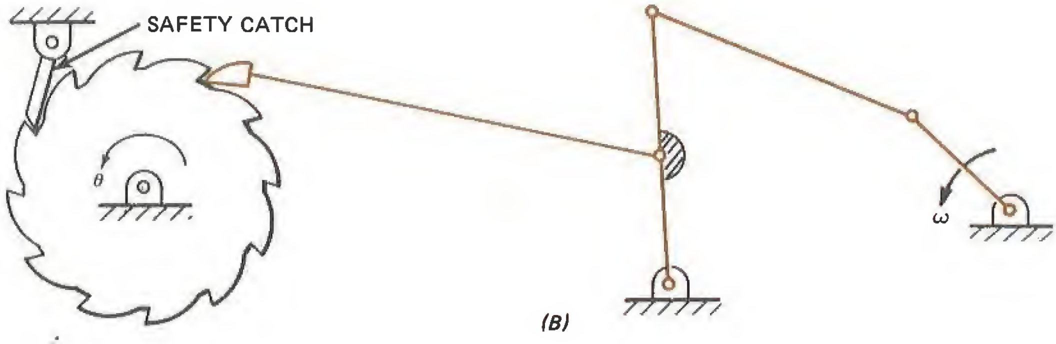


Fig. 15-18 (B) Ratchet Intermittent Motion

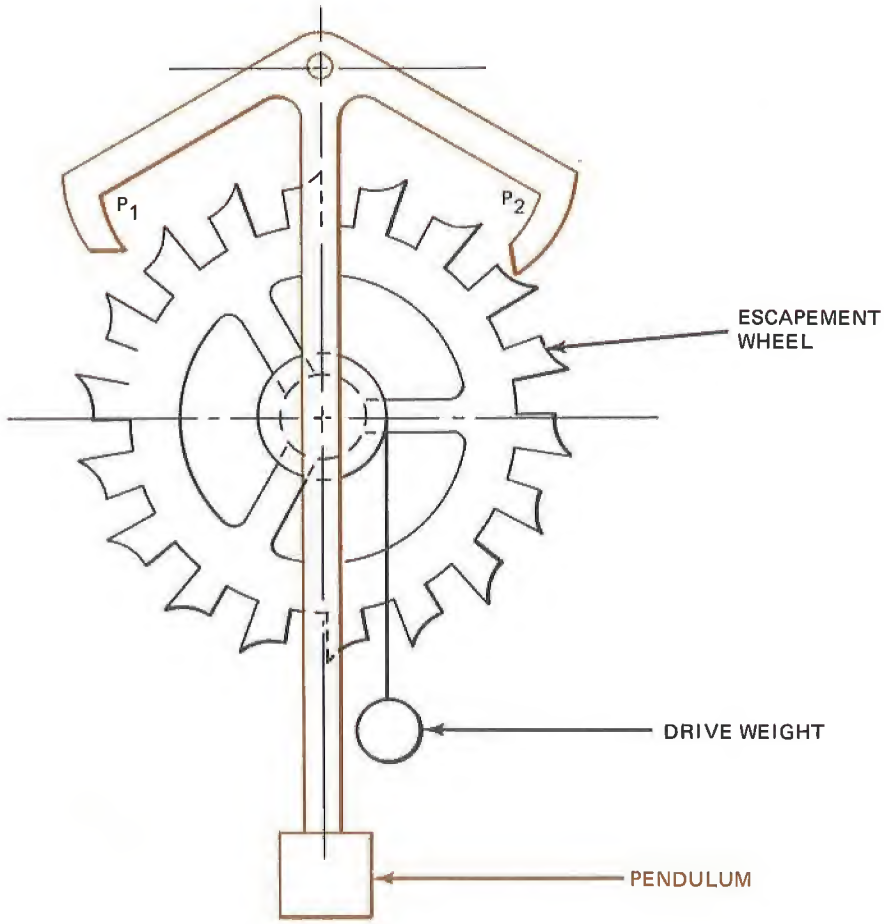


Fig. 15-18 (C) Escapement Mechanism

MATERIALS

- 1 Engineer's scale
- 1 Straightedge
- 1 Compass
- 1 Divider
- 1 Protractor
- 1 Drafting pencil

PROCEDURE

1. Find the displacement curve for point P in the linkage of figure 15-19. You may make *one* simplifying assumption if desired.

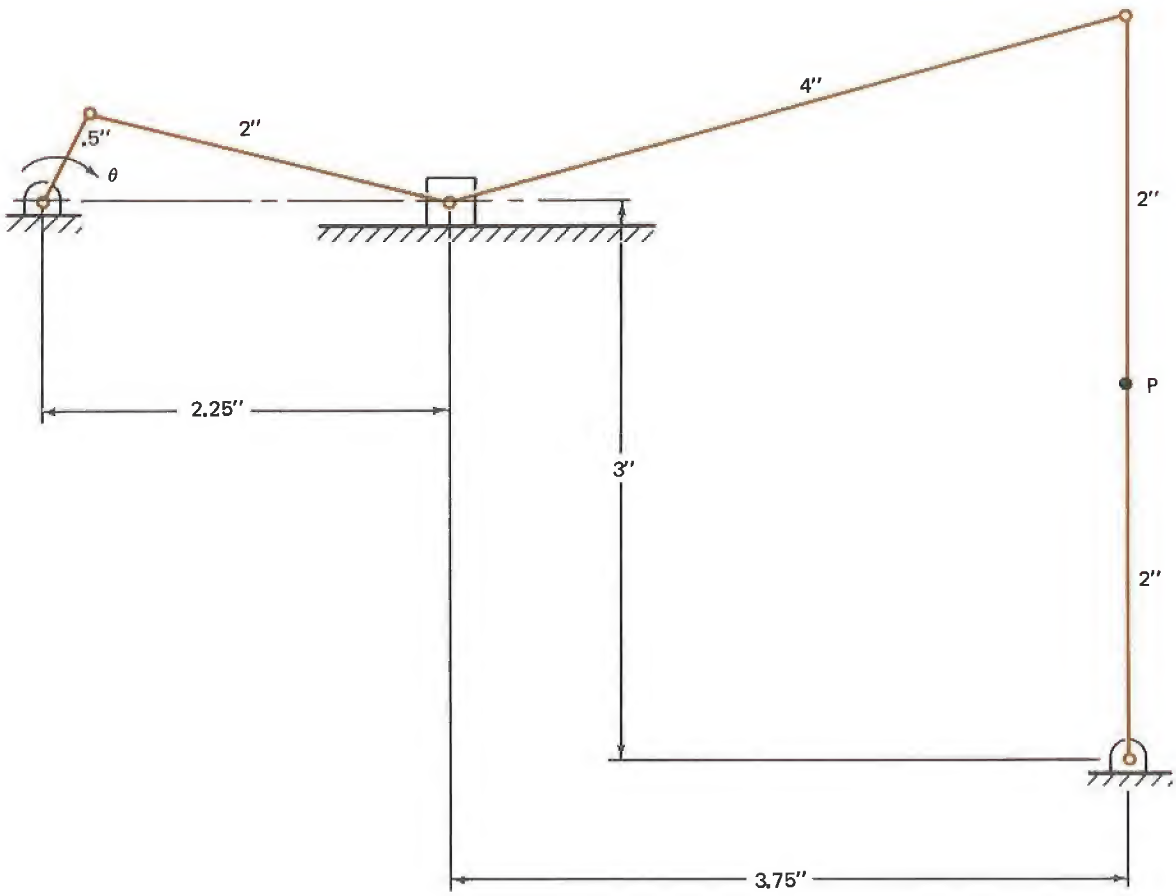


Fig. 15-19 Locus of Link Points

2. Graphically differentiate the displacement curve from problem 1 and find the velocity, acceleration, and jerk curves for point P.

3. Find the instantaneous centers of rotation for links 1, 2, and 3 in figure 15-20 for the position shown.

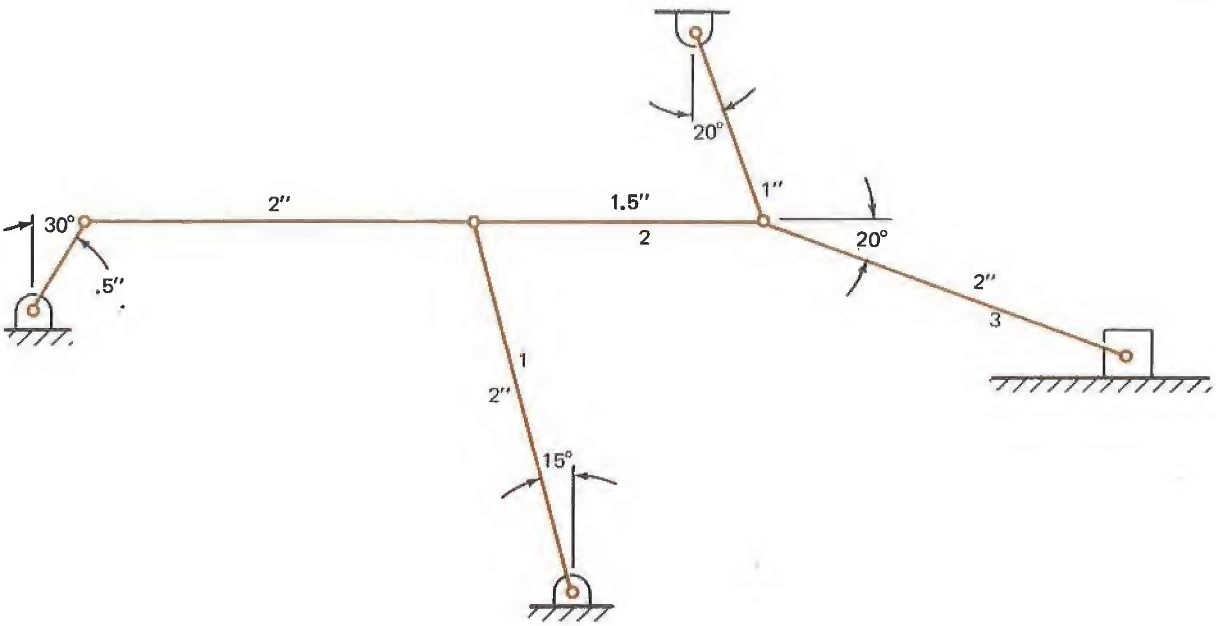


Fig. 15-20 Instantaneous Centers

4. Find the velocity polygon for point P in figure 15-21.

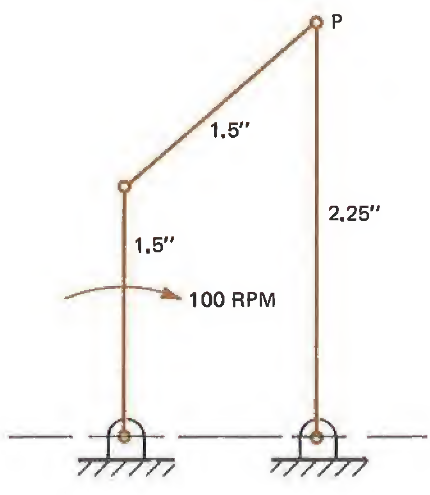


Fig. 15-21 Velocity Polygon

5. Draw and lable a straight-line mechanism which will perform the operations $S = 2(S_1 + S_2)$.
6. Find the approximate ratio of F_{out}/F_{in} for the toggle mechanism in figure 15-22, for input lever movement from 10° to 5° CCW from the positive X-axis.
7. Draw a single pinned link which will, with two or more scales added, perform the operations $S = 2.73S_1$ and $S = S_1/4.3$.

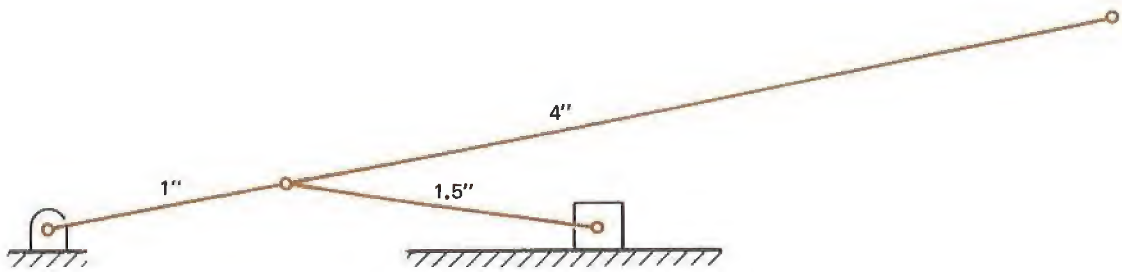


Fig. 15-22 Toggle Mechanism

8. Find the profile of a single-follower cam which will give the displacement curve of figure 15-23.

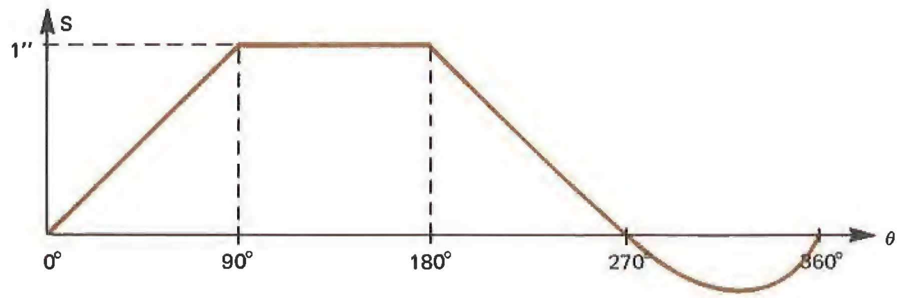


Fig. 15-23 Single-Follower Cam Curve

9. Sketch a double-follower cam linkage with a base circle diameter of 1 in., which will give the displacement curve of figure 15-23 at some point.

10. Sketch a gear train mechanism which has an input gear of d = 2 in., ω = 100 RPM, and the output desired is 250 RPM.

ANALYSIS GUIDE. What do you think caused the major errors in each of the procedure steps? You should recall the sources of error you have described in previous exercises.

PROBLEMS

1. Why is jerk (da/dt) an important quantity in describing the motion of a mechanism.
2. If you double the S scale on a displacement curve, what will the velocity at some time, t; be?
3. Why should high speed mechanisms have third or high-order displacement curves whenever possible?
4. Why are cams (with single radial followers) which generate *high* velocities normally impractical?
5. What is the single most important limitation of an external Geneva wheel mechanism from a kinematics standpoint?

EXPERIMENT 1 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

EXPERIMENT 1 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

EXPERIMENT 2 __________ Name __________

Date: __________ Class __________ Instructor __________

EXPERIMENT 2 ____________ Name ________________________________

Date: ____________________ Class __________ Instructor ______________

EXPERIMENT 3 Name ______________________

Date: ______________ Class ______________ Instructor ______________

EXPERIMENT 3

Name ______________________

Date: ______________ Class __________ Instructor ______________

EXPERIMENT 4

Name ______________________

Date: ______________ Class ________ Instructor ______________

EXPERIMENT 4 Name ______________________________

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EXPERIMENT 5

Name ______________________

Date: ______________________

Class ____________ Instructor ______________

EXPERIMENT 5 Name ______________________

Date: ______________ Class __________ Instructor ______________

EXPERIMENT 6

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EXPERIMENT 7

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EXPERIMENT 8 Name ______________________

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EXPERIMENT 9 Name ____________________

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EXPERIMENT 10 ______________ Name ______________________________

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EXPERIMENT 10 ____________ Name ______________________________

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EXPERIMENT 11

Name ______________________

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Class __________ Instructor ______________

EXPERIMENT 11 Name ______________________

Date: ______________ Class __________ Instructor ______________

EXPERIMENT 12 Name ______

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Date: ______________ Class ________ Instructor ______________

EXPERIMENT 13 Name ______________________

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EXPERIMENT 14 Name ______________________

Date: ______________ Class ________ Instructor ____________

EXPERIMENT 14

Name ______________________________

Date: ____________________ Class __________ Instructor ____________________

EXPERIMENT 15 Name ______

Date: ______ Class ______ Instructor ______

EXPERIMENT 15 Name ______________________

Date: ______________ Class ________ Instructor ______________